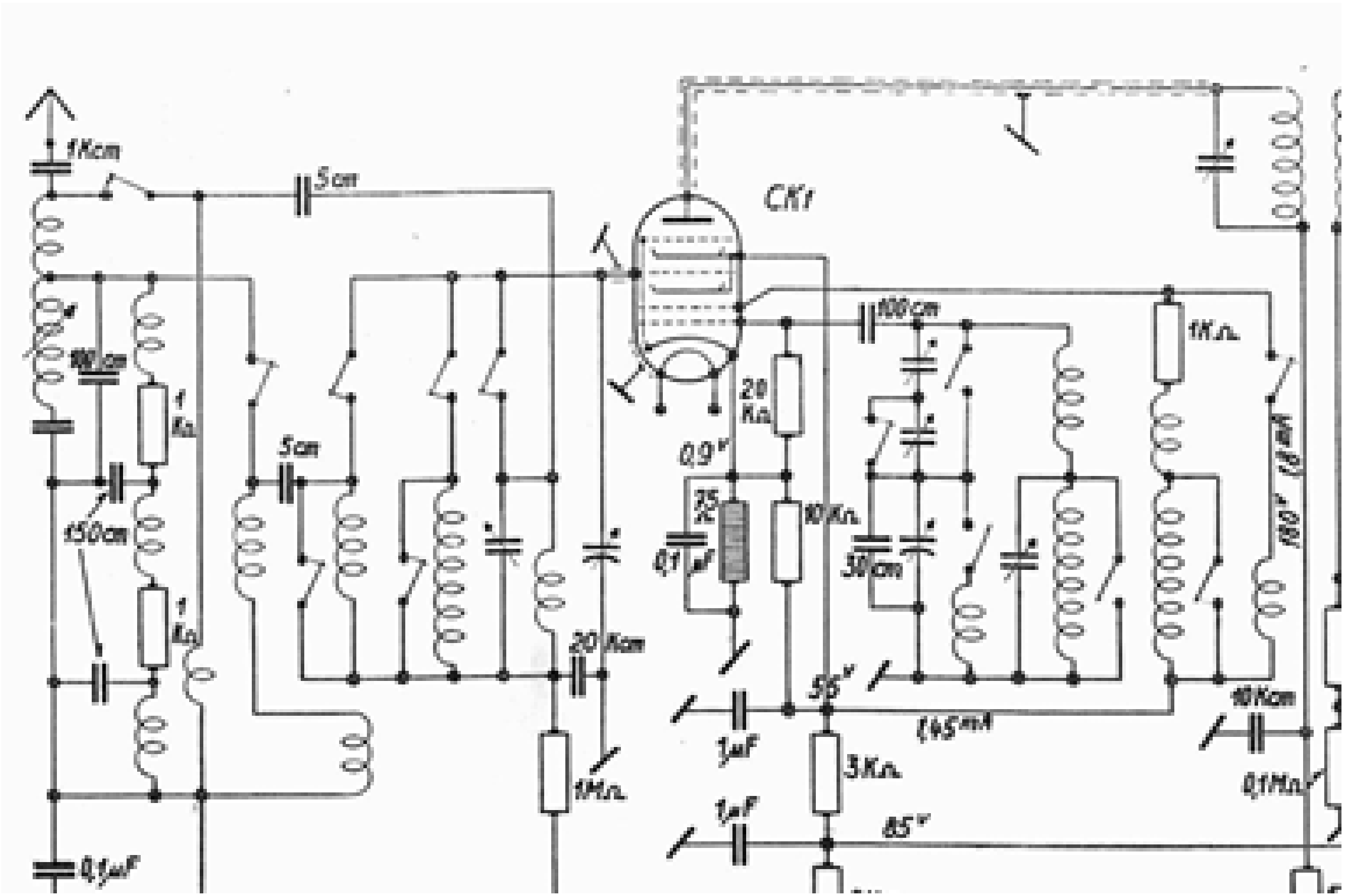
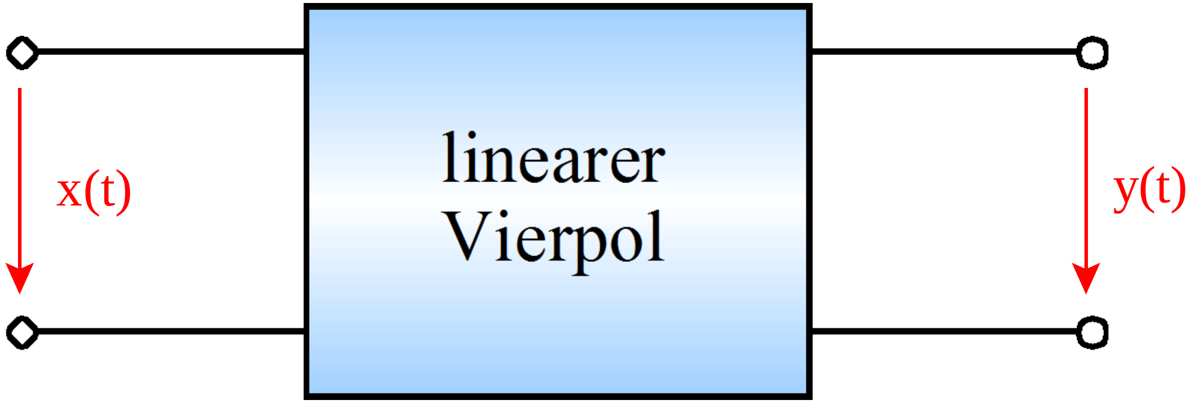
7. Ausgewählte Wechselstromanordnungen



7. Ausgewählte Wechselstromanordnungen

7.1 Schaltungen mit frequenzselektiven Eigenschaften



Zeitbereich: $y(t) = f\left(x(t), \frac{dx}{dt}, \int x(t)dt\right)$

Bildbereich (komplexe Ebene): $\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega)\,\underline{\hat{X}}(j\omega)$

Übertragungsfunktion: $$\underline{H}(j\omega) = \frac{\underline{\hat{Y}}(j\omega)}{\underline{\hat{X}}(j\omega)}$$

$$\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega)\,\underline{\hat{X}}(j\omega)$$

$$\hat{Y}(\omega)\,e^{j\varphi_y(\omega)} = H(\omega)\,e^{j\varphi_H(\omega)}\,\hat{X}(\omega)\,e^{j\varphi_x(\omega)}$$

Amplitudenübertragung: $\hat{Y}(\omega) = H(\omega)\,\hat{X}(\omega)$

Amplitudenfrequenzgang des Systems

Phasenübertragung: $\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)$

Phasenfrequenzgang des Systems

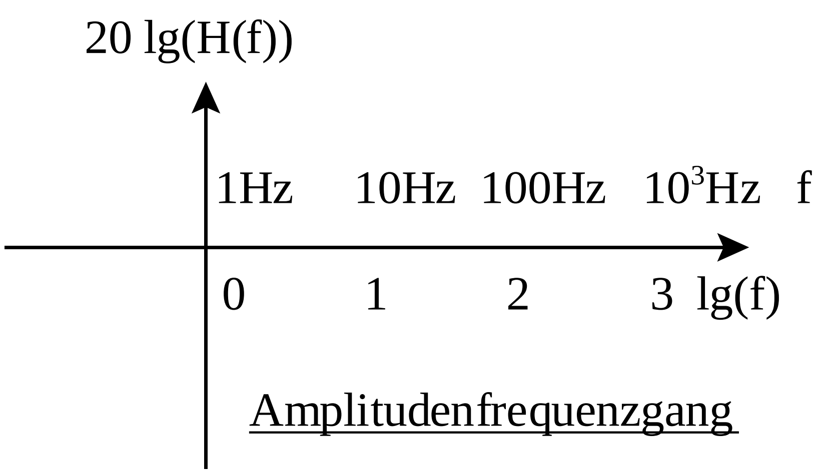
Amplitudenübertragung: $\hat{Y}(\omega) = H(\omega)\ \hat{X}(\omega)$

Phasenübertragung: $\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)$

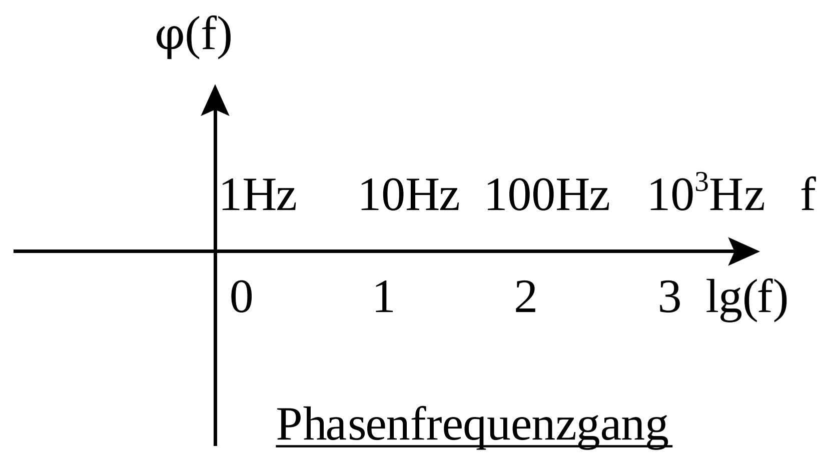
$$\lg\left(\hat{Y}(\omega)\right) = \lg\left(H(\omega)\right) + \lg\left(\hat{X}(\omega)\right)$$

$$20\lg\left(\hat{Y}(\omega)\right) = 20\lg\left(H(\omega)\right) + 20\lg\left(\hat{X}(\omega)\right) \quad \text{in dB}$$

- übliche grafische Darstellung:



Amplitudenfrequenzgang

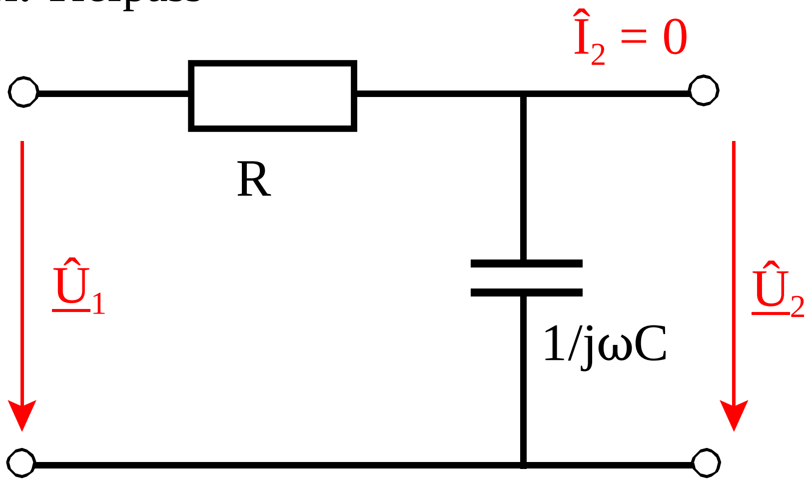


Phasenfrequenzgang

7.7.1 RC- und RL- Schaltungen

a) RC-Tiefpass

Beispiel: Tiefpass



RC - Glied als Tiefpaß

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{1}{1 + j\omega CR}$$

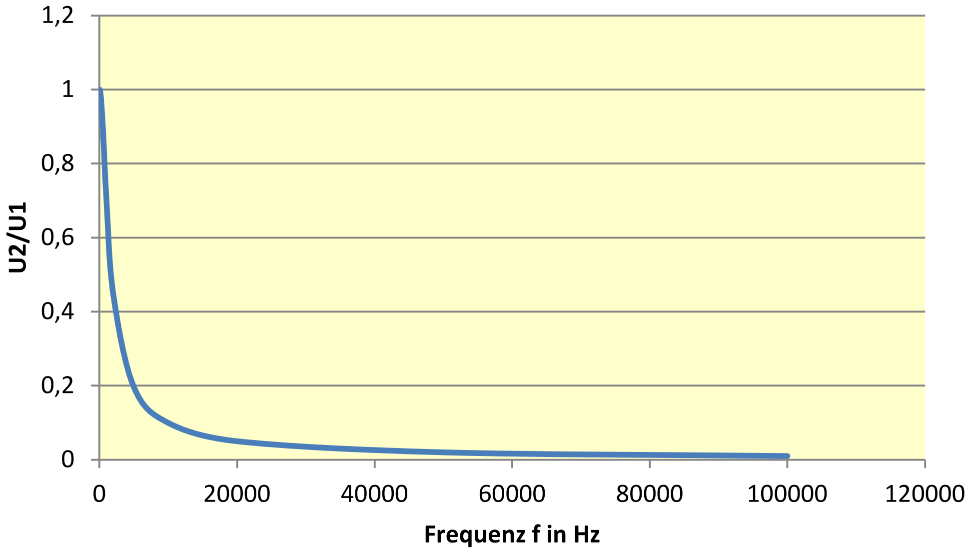
<u>Amplitudenfrequenzgang:</u>

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

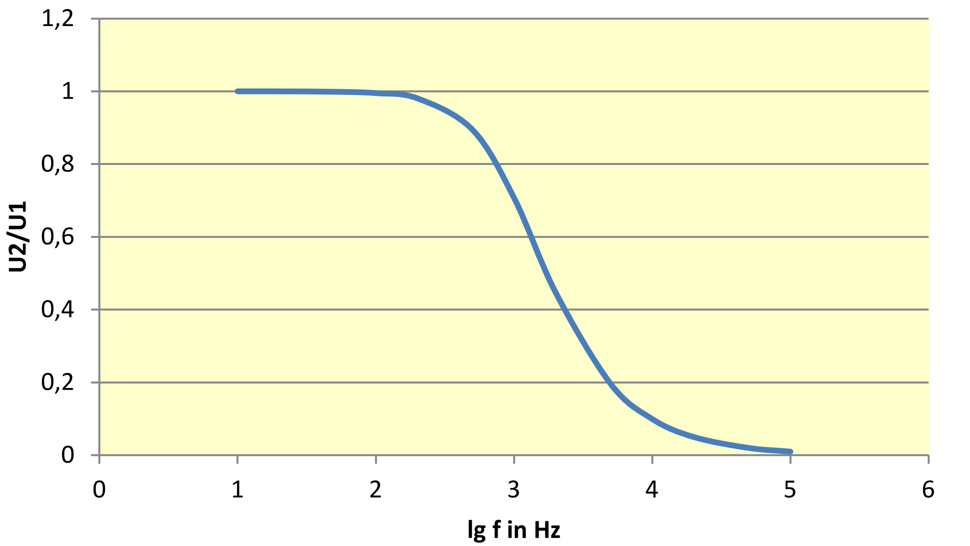
<u>Phasenfrequenzgang:</u>

$$\varphi_H(\omega) = \varphi_{u2}(\omega) - \varphi_{u1}(\omega) = -\arctan \omega CR$$

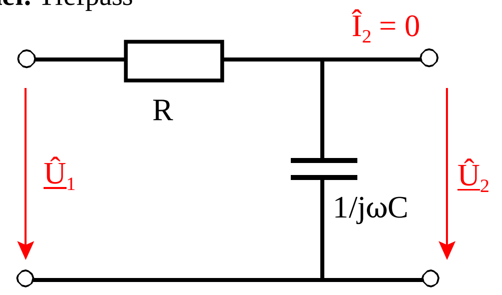
Lineare Darstellung des Amplitudengangs



Logarithmische Darstellung des Amplitudengangs



Beispiel: Tiefpass

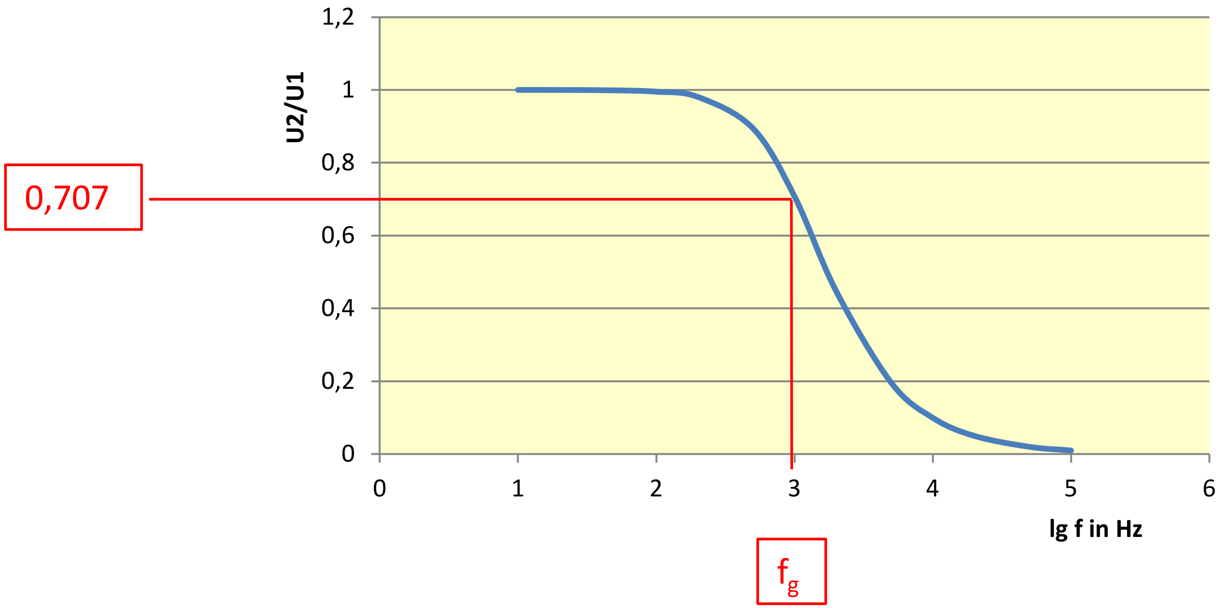


RC - Glied als Tiefpaß

Werte:
R=159 Ω
C=1 µF

Amplitudenfrequenzgang:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1+(\omega\, CR)^2}}$$



Grenzfrequenz:

ω_g oder f_g mit $\omega_g=2\pi f_g$

Definition: f_g ist jene Frequenz, bei der $|H(\omega)|$ auf den $\frac{1}{\sqrt{2}} = 2^{-\frac{1}{2}}$ - fachen Wert

des Maximalwertes abgesunken ist oder
der Phasenwinkel $\varphi(\omega)$= 45° beträgt oder
der Realteil von $\underline{H}(j\omega)$ = Imaginärteil von $\underline{H}(j\omega)$ ist.

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{1}{1+ j\omega CR}$$ => $\omega_g RC=1$ oder $$\omega_g = \frac{1}{RC} = 2 \cdot \pi \cdot f_g = \frac{1}{\tau}$$

Amplitudenfrequenzgang:

Grafische Darstellung des Amplitudengangs mit Hilfsgeraden:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

$$20\lg\left(\hat{Y}(\omega)\right) = 20\lg\left(H(\omega)\right) + 20\lg\left(\hat{X}(\omega)\right) \quad \text{in dB}$$

$$20\cdot\lg\left[H(\omega)\right] = -10\cdot\lg\left[1+(\omega RC)^2\right] = -10\cdot\lg\left[1+\left(\frac{\omega}{\omega_g}\right)^2\right]$$

$$\omega_g = \frac{1}{RC}$$

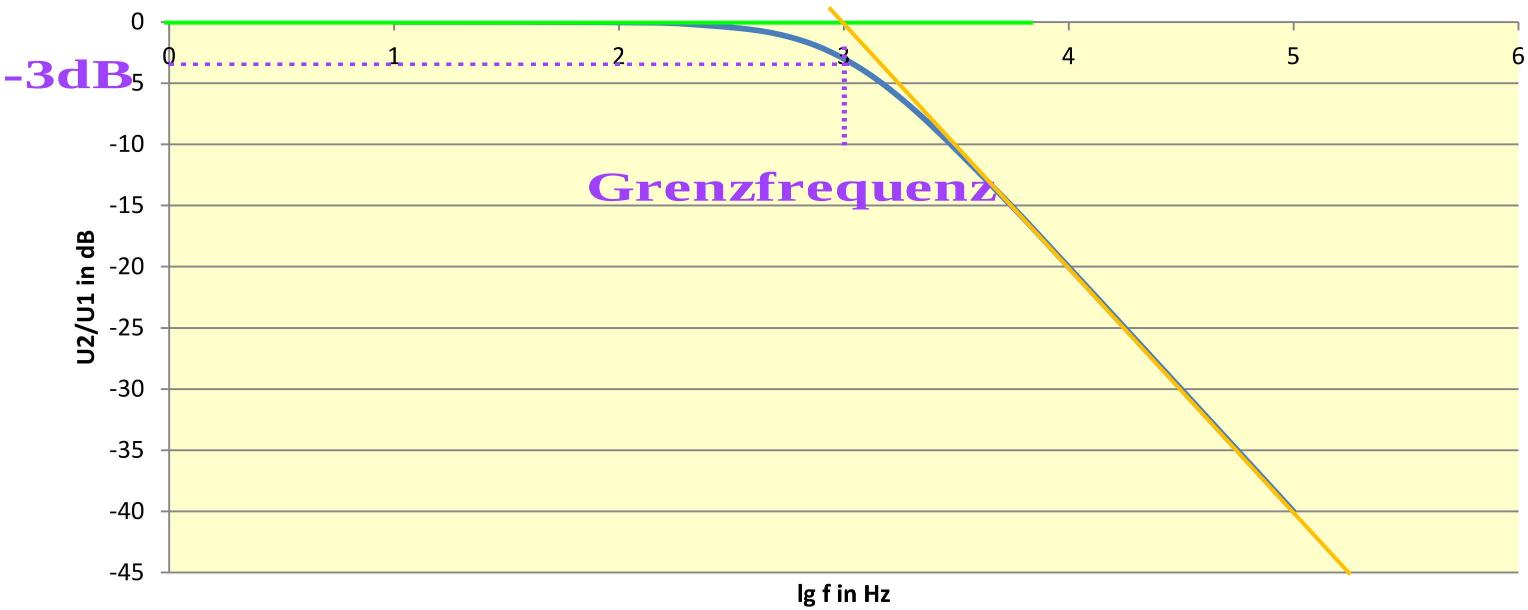
oder $$20\cdot\lg\left[H(f)\right] = -10\cdot\lg\left[1+\left(\frac{f}{f_g}\right)^2\right]$$

$$\text{für } \frac{f}{f_g} << 1 \text{ gilt: } 20\cdot\lg\left[H(f)\right] = -10\cdot\lg\left[1+\left(\frac{f}{f_g}\right)^2\right] \approx 0$$

$$\text{für } \frac{f}{f_g} >> 1 \text{ gilt: } 20\cdot\lg\left[H(f)\right] \approx -20\cdot\lg\left[\left(\frac{f}{f_g}\right)\right] = -20\cdot\lg f + 20\cdot\lg f_g$$

$$\frac{f}{f_g} << 1 \quad 20 \cdot \lg[H(f)] \approx 0$$

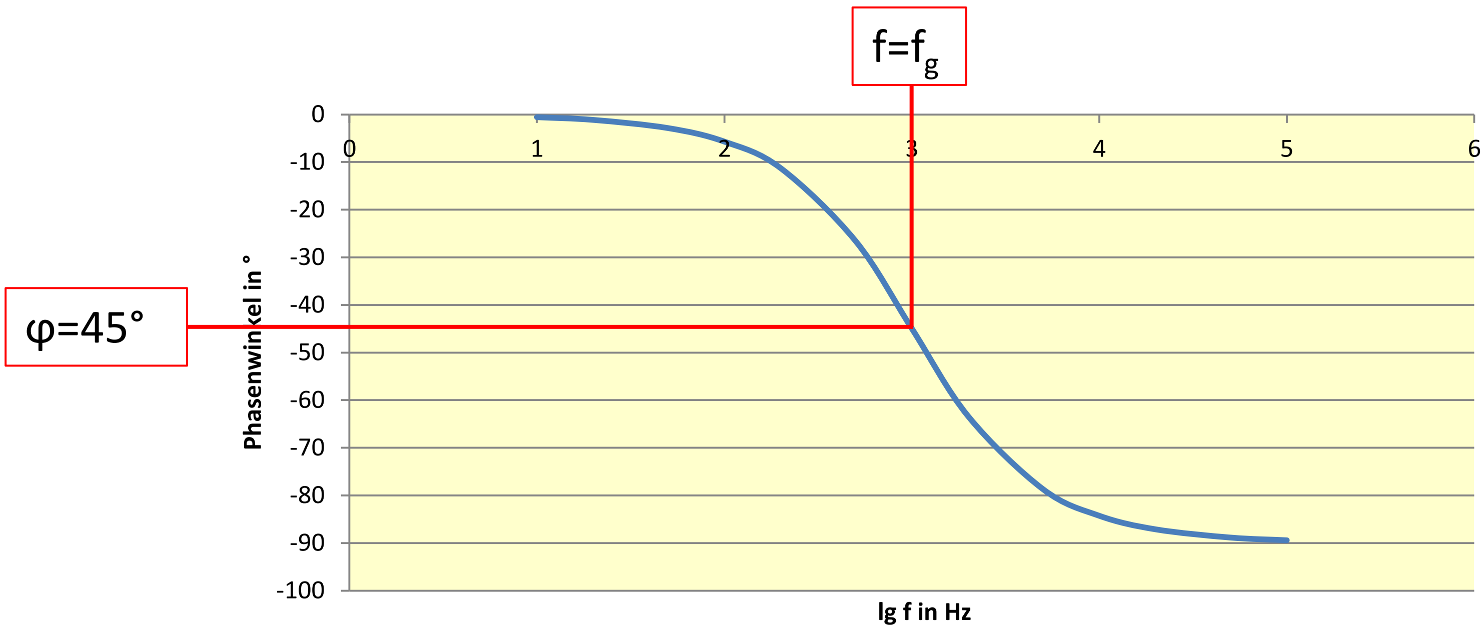
$$\frac{f}{f_g} >> 1 \quad 20 \cdot \lg[H(f)] \approx -20 \cdot \lg f + 20 \cdot \lg f_g$$



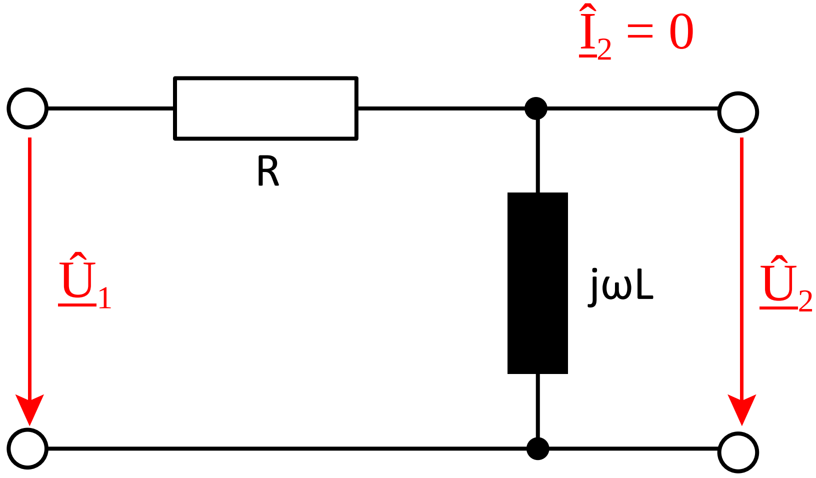
Grafische Darstellung des Phasengangs: $\varphi_H(\omega) = -\arctan \omega CR$

$$\omega_g = \frac{1}{RC}$$

$$\varphi_H(\omega) = -\arctan\left(\frac{\omega}{\omega_g}\right) \quad \text{oder} \quad \varphi_H(f) = -\arctan\left(\frac{f}{f_g}\right)$$



b) R-L-Hochpass:



$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{j\omega L}{R + j\omega L}$$

$$= \frac{1}{1 + \frac{R}{j\omega L}} = \frac{1}{1 - j\frac{R}{\omega L}}$$

Amplitudengang:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$

Grenzfrequenz:

$$\frac{R}{\omega_g L} = 1 \qquad \omega_g = \frac{R}{L} = \frac{1}{\tau}$$

Phasengang:

$$\varphi_H(\omega) = \varphi_{U2}(\omega) - \varphi_{U1}(\omega)$$

$$\varphi_H(\omega) = 0 + \arctan\left(\frac{R}{\omega L}\right)$$

Grafische Darstellung des Amplitudengangs mit Hilfsgeraden:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$

$$20\lg\left(\hat{Y}(\omega)\right) = 20\lg\left(H(\omega)\right) + 20\lg\left(\hat{X}(\omega)\right) \quad \text{in dB}$$

$$20 \cdot \lg\left[H(\omega)\right] = -10 \cdot \lg\left[1 + \left(\frac{R}{\omega L}\right)^2\right] = -10 \cdot \lg\left[1 + \left(\frac{\omega_g}{\omega}\right)^2\right]$$

$$\omega_g = \frac{R}{L}$$

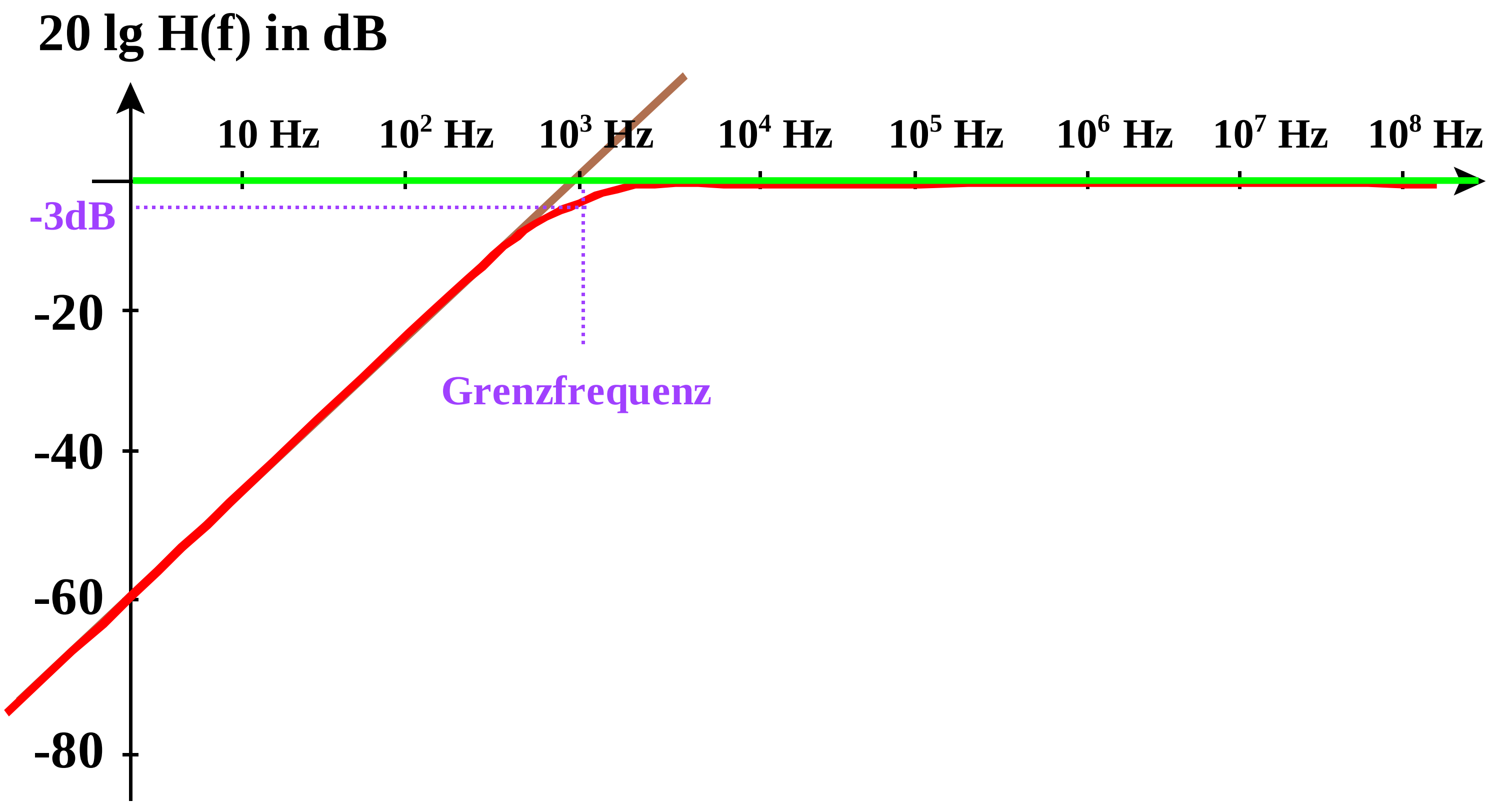
oder $$20 \cdot \lg\left[H(f)\right] = -10 \cdot \lg\left[1 + \left(\frac{f_g}{f}\right)^2\right]$$

$$\text{für } \frac{f_g}{f} << 1 \text{ gilt: } 20 \cdot \lg\left[H(f)\right] = -10 \cdot \lg\left[1 + \left(\frac{f_g}{f}\right)^2\right] \approx 0$$

$$\text{für } \frac{f_g}{f} >> 1 \text{ gilt: } 20 \cdot \lg\left[H(f)\right] \approx -20 \cdot \lg\left[\left(\frac{f_g}{f}\right)\right] = +20 \cdot \lg f - 20 \cdot \lg f_g$$

$$\frac{f_g}{f} << 1: \quad 20 \cdot \lg\left[H(f)\right] \approx 0$$

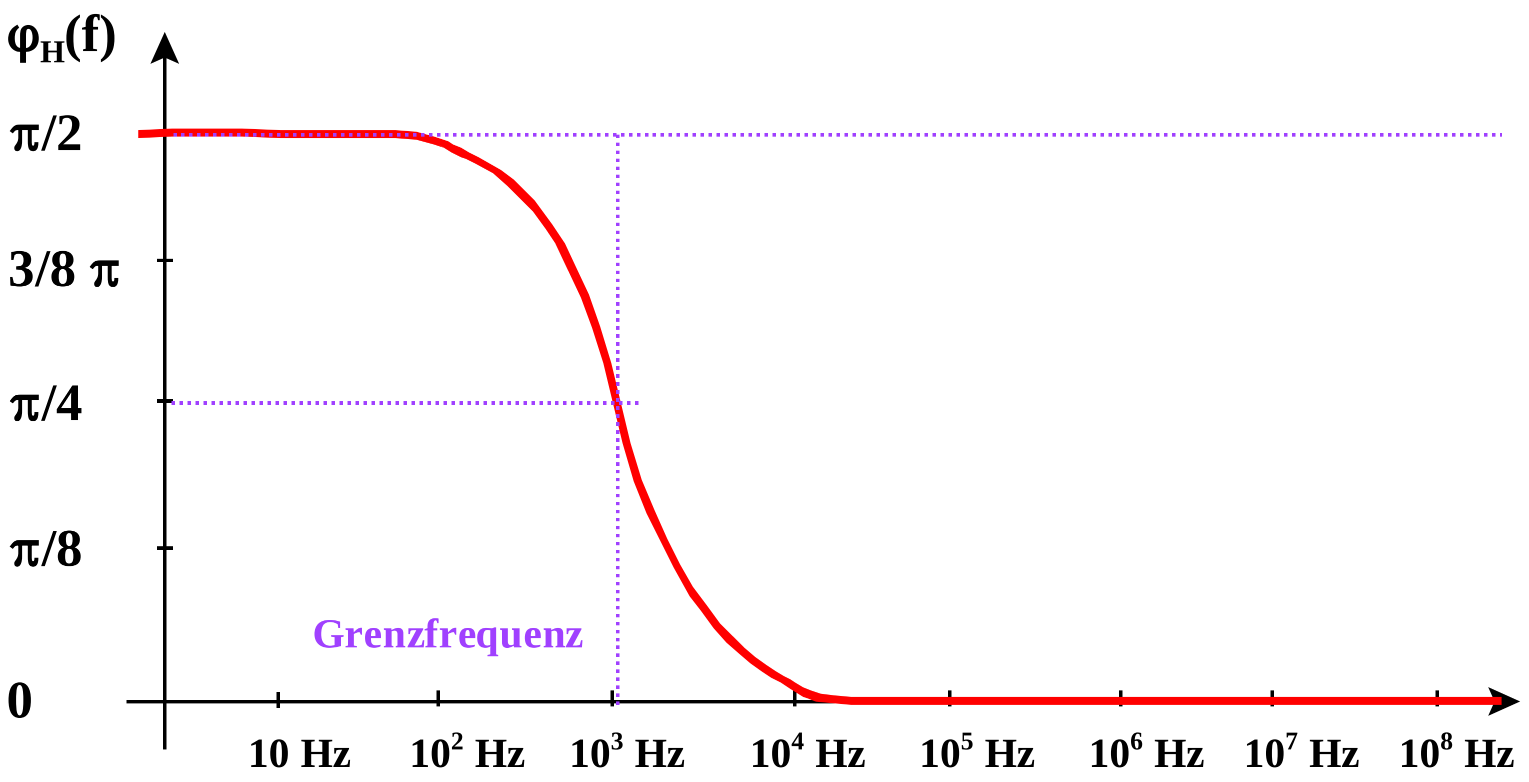
$$\frac{f_g}{f} >> 1: \quad 20 \cdot \lg\left[H(f)\right] \approx +20 \cdot \lg f - 20 \cdot \lg f_g$$



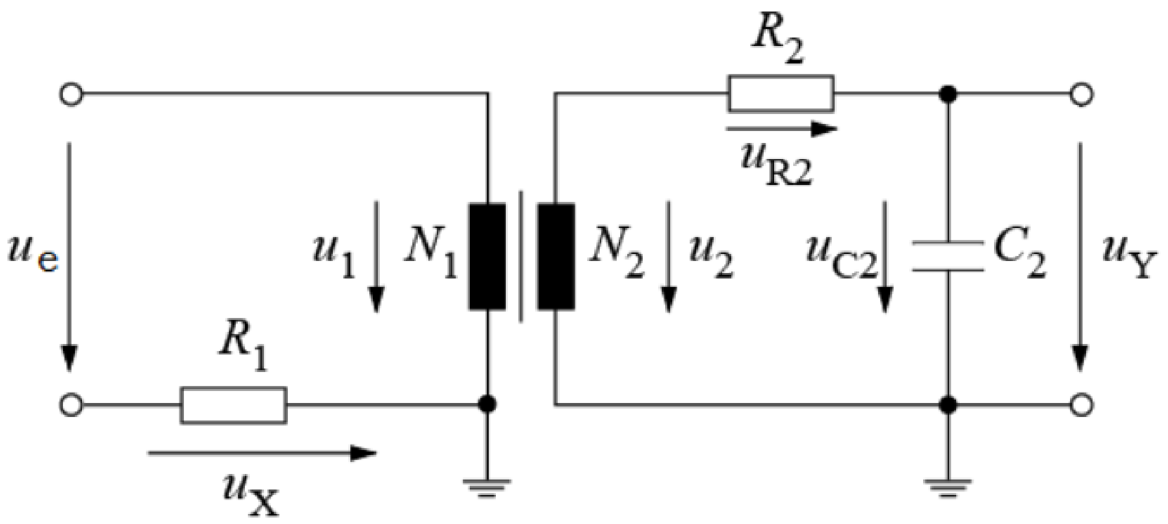
Grafische Darstellung des Phasengangs:

$$\varphi_H(\omega) = 0 + \arctan\left(\frac{R}{\omega L}\right)$$

$$\varphi_H(\omega) = \arctan\left(\frac{\omega_g}{\omega}\right) \quad \text{oder} \quad \varphi_H(f) = \arctan\left(\frac{f_g}{f}\right) \qquad \omega_g = \frac{R}{L}$$



c) RC-Tiefpass als Integrierglied



Messwiderstand R_1=100Ω, R_2=30kΩ, C_2=4,7µF,
N_1=2600, N_2=90, l_{Fe}=13cm, A_{Fe}=3,1cm²

Aufnahme der B-H-Kennlinie des Eisenkerns, dazu Erfassung der Feldstärke H über Strom durch Primärwicklung N1:

$$H = \frac{N_1 \cdot U_X}{l_{Fe} \cdot R_1}$$

B hat differenziellen Zusammenhang:

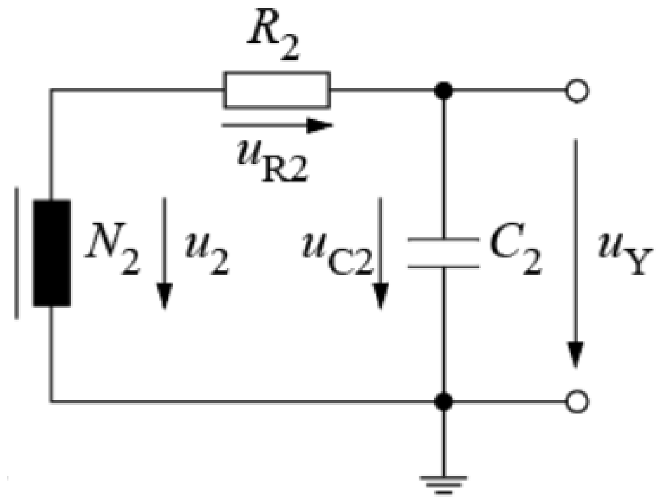
$$u_2 = \frac{N_2 \cdot d\Phi}{dt} = \frac{N_2 \cdot A_{Fe} \cdot dB}{dt}$$

$$dB = \frac{u_2(t)}{N_2 \cdot A_{Fe}} \cdot dt$$

$$B = \frac{1}{N_2 \cdot A_{Fe}} \int u_2(t) \cdot dt$$

$$dB = \frac{u_2(t)}{N_2 \cdot A_{Fe}} \cdot dt$$

$$B = \frac{1}{N_2 \cdot A_{Fe}} \int u_2(t) \cdot dt$$



$u_2 = u_{R2} + u_{C2}$ wenn keine Last angeschlossen, gilt $i_{R2} = i_{C2} = i$

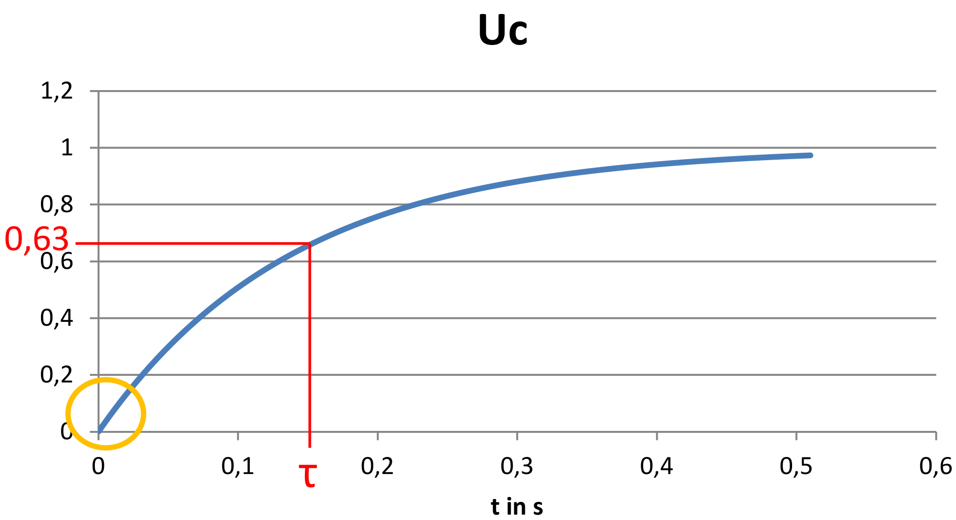
$u_2 = i \cdot R_2 + u_{C2}$ wenn $u_{R2} >> u_{C2}$ (also $R_2 >> \frac{1}{\omega \cdot C}$)

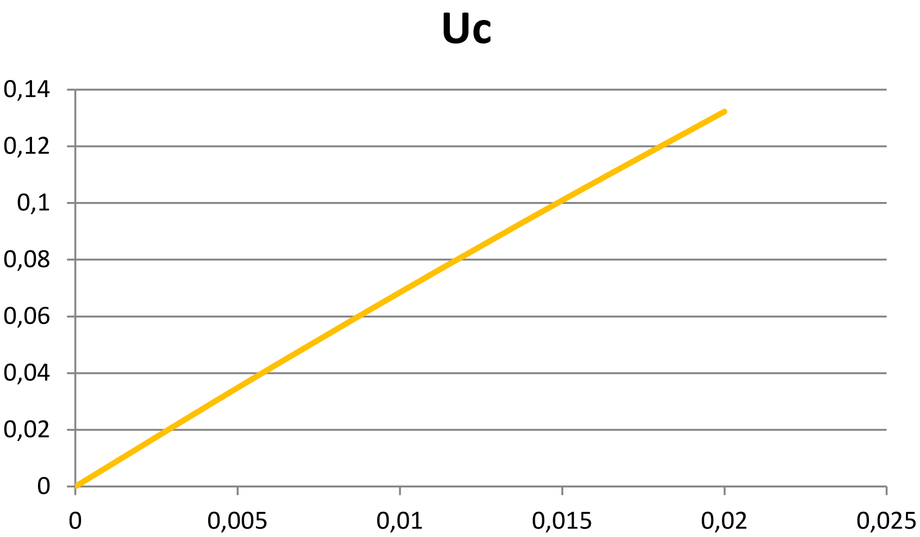
$u_2 = i \cdot R_2$ $\quad i = C_2 \cdot \frac{du_{C2}}{dt}$

$u_{R2} = R_2 \cdot C_2 \cdot \frac{du_{C2}}{dt}$ $\quad du_{C2} = \frac{1}{R_2 \cdot C_2} u_{R2} \cdot dt$

$$u_{C2} = \frac{1}{R_2 \cdot C_2} \int u_{R2} \cdot dt$$

$$u_{C2} = \frac{1}{R_2 \cdot C_2} \int u_{R2} \cdot dt$$

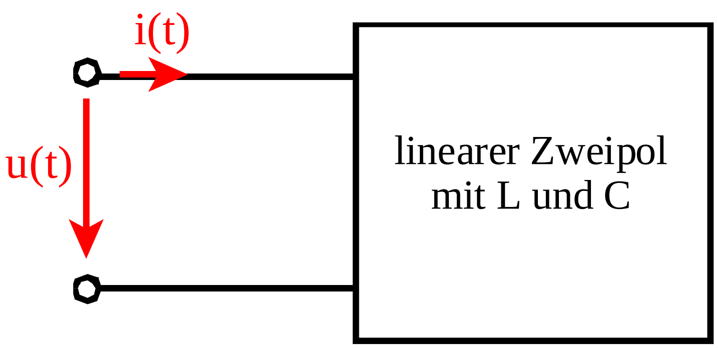




$$\tau = R_2 \cdot C_2 = 30k\Omega \cdot 4{,}7\mu F = 0{,}141s$$

Integrationszeit t_i in GET2:
f=50 Hz => T=1/f=0,02s,
also T<<τ und $u_c << u_R$

Die Resonanzbedingungen

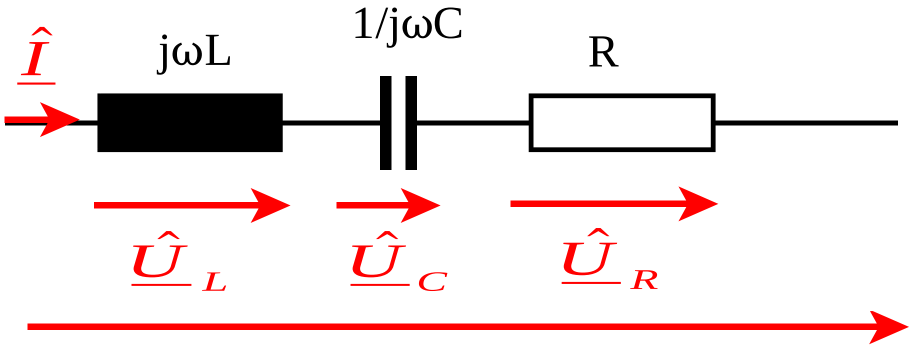


Ein linearer Zweipol befindet sich dann in Resonanz, wenn er sich trotz Vorhandensein von mindestens einer Induktivität und mindestens einer Kapazität bei Wechselstrom wie ein Ohmscher Widerstand verhält, wenn also nur die "verbrauchte" Wirkleistung von der äußeren Schaltung nachgeliefert werden muß.

$$\mathrm{Im}\left\{\underline{Z}\right\} = 0 \qquad \mathrm{Im}\left\{\underline{Y}\right\} = 0 \qquad \varphi_{\mathrm{UI}} = \varphi_{\mathrm{U}} - \varphi_{\mathrm{I}} = 0$$

Tacomabrücke

Die Reihen- oder Spannungsresonanz



$$\underline{\hat{U}} = \hat{U}e^{j\varphi_U}$$

$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Bestimmung der Resonanzfrequenz

$$\operatorname{Im}\left\{\underline{Z}\left(\omega_0\right)\right\} = 0$$

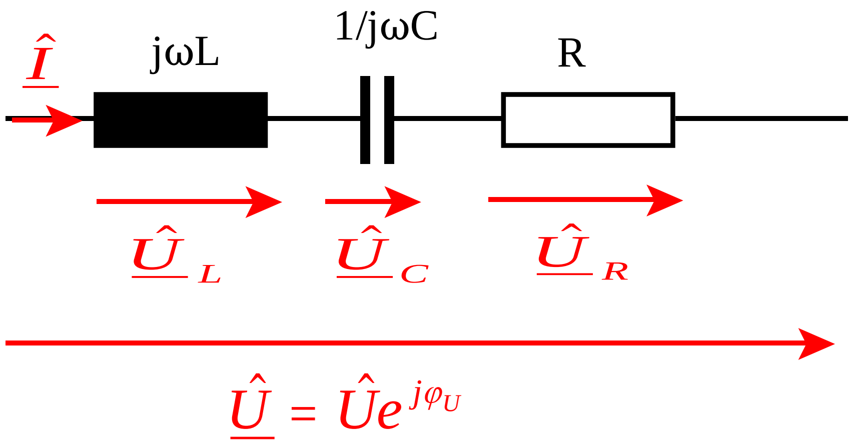
$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\omega_0^2 LC - 1 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Thomsonsche Schwingungsformel



$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

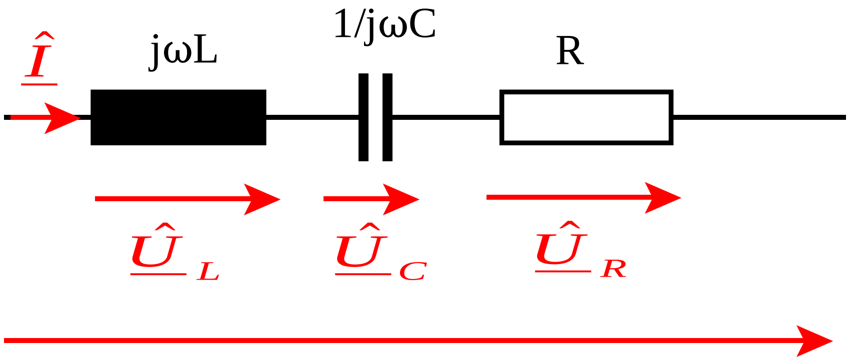
Die Abhängigkeit des Stromes von der Frequenz

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\left(\frac{\omega}{\omega_0}\omega_0 L - \frac{1}{\frac{\omega}{\omega_0}\omega_0 C}\right)}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\omega_0 L\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R\left[1 + j\frac{\omega_0 L}{R}\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)\right]}$$



$$\underline{\hat{U}} = \hat{U} e^{j\varphi_U}$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

mit den Abkürzungen

$$\frac{\omega}{\omega_0} = \frac{f}{f_0} = x$$ normierte Frequenz

$$\frac{\omega_0 L}{R} = Q_R$$ Güte $$Q_R = \frac{\text{Leistung an } L \text{ oder } C}{\text{Leistung an } R}$$

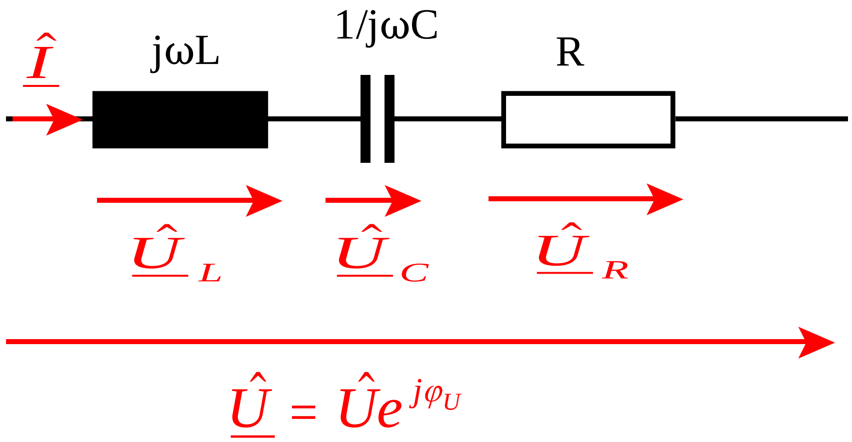
$$\underline{\hat{I}}_0 = \frac{\underline{\hat{U}}}{R}$$ Strom im Resonanzfall

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R\left[1 + j\frac{\omega_0 L}{R}\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)\right]}$$

entsteht

$$\underline{\hat{I}} = \frac{\underline{\hat{I}}_0}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

$$\frac{\underline{\hat{I}}}{\underline{\hat{I}}_0} = \frac{1}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$



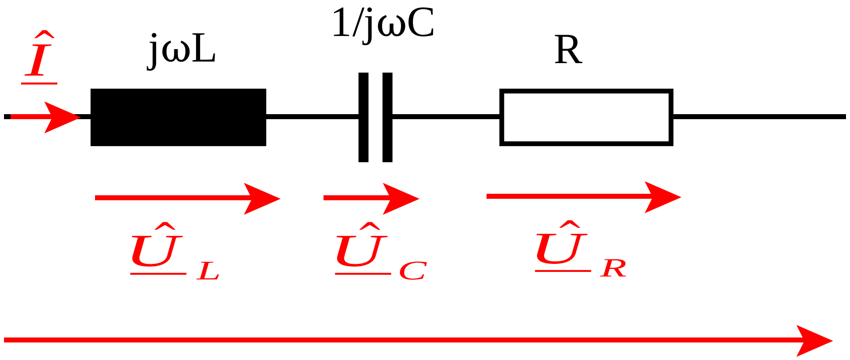
$$\frac{\underline{\hat{I}}}{\underline{\hat{I}}_0} = \frac{1}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

$$\left|\frac{\underline{\hat{I}}}{\underline{\hat{I}}_0}\right| = \frac{1}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

Betrag des Stromes

$$\varphi_{I,I_0} = -\arctan\left(Q_R\left(x - \frac{1}{x}\right)\right)$$

Phasenlage des Stromes



$$\underline{\hat{U}} = \hat{U} e^{j\varphi_U}$$

Berechnung der Spannung am Widerstand
in Abhängigkeit von der Frequenz

$$\underline{\hat{U}}_R = R\underline{\hat{I}}$$

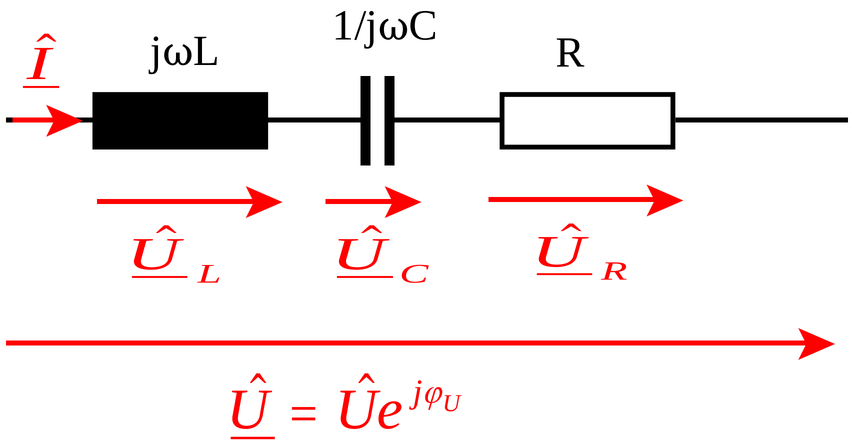
$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{I}}_0}{1 + jQ_R\left(x - \frac{1}{x}\right)} \qquad \underline{\hat{I}}_0 = \frac{\underline{\hat{U}}}{R}$$

$$\underline{\hat{U}}_R = \frac{R\underline{\hat{U}}}{R\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\underline{\hat{U}}_R = \frac{\underline{\hat{U}}}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

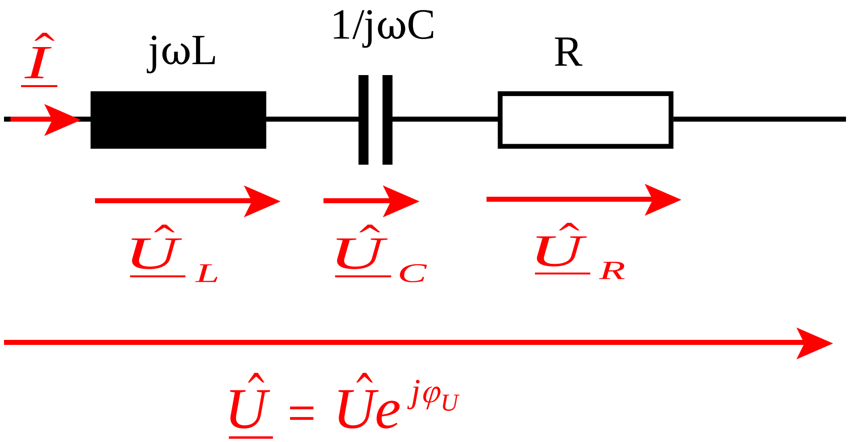
$$\frac{\underline{\hat{U}}_R}{\underline{\hat{U}}} = \frac{1}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$



$$\frac{\underline{\hat{U}}_R}{\underline{\hat{U}}} = \frac{1}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

$$\left|\frac{\underline{\hat{U}}_R}{\underline{\hat{U}}}\right| = \frac{1}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

$$\varphi_{U_R,U} = -\arctan\left(Q_R\left(x - \frac{1}{x}\right)\right)$$



$$\hat{\underline{I}} = \frac{\hat{\underline{U}}}{R\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

Berechnung der Spannung an der Induktivität in Abhängigkeit von der Frequenz

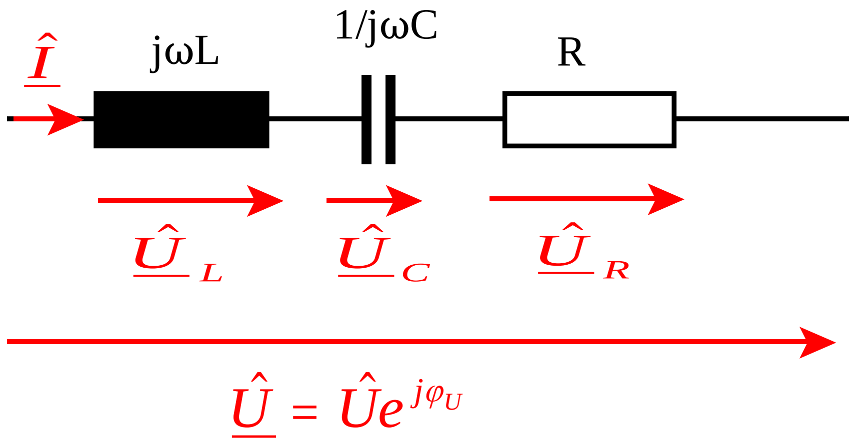
$$\hat{\underline{U}}_L = j\omega L\hat{\underline{I}} = j\frac{\omega}{\omega_0}\omega_0 L\hat{\underline{I}}$$

$$\hat{\underline{U}}_L = \frac{xj\omega_0 L\hat{\underline{U}}}{R\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\frac{\omega_0 L}{R} = Q_R$$

$$\hat{\underline{U}}_L = \frac{xjQ_R\hat{\underline{U}}}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

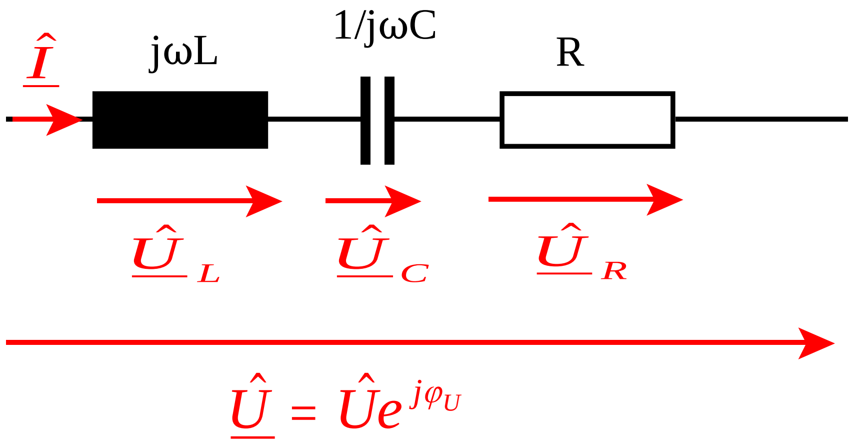
$$\frac{\hat{\underline{U}}_L}{\hat{\underline{U}}} = \frac{xjQ_R}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$



$$\frac{\underline{\hat{U}}_L}{\underline{\hat{U}}} = \frac{xjQ_R}{1 + jQ_R\left(x - \frac{1}{x}\right)}$$

$$\left|\frac{\underline{\hat{U}}_L}{\underline{\hat{U}}}\right| = \frac{xQ_R}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

$$\varphi_{U_L,U} = \frac{\pi}{2} - \arctan\left(Q_R\left(x - \frac{1}{x}\right)\right)$$



$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

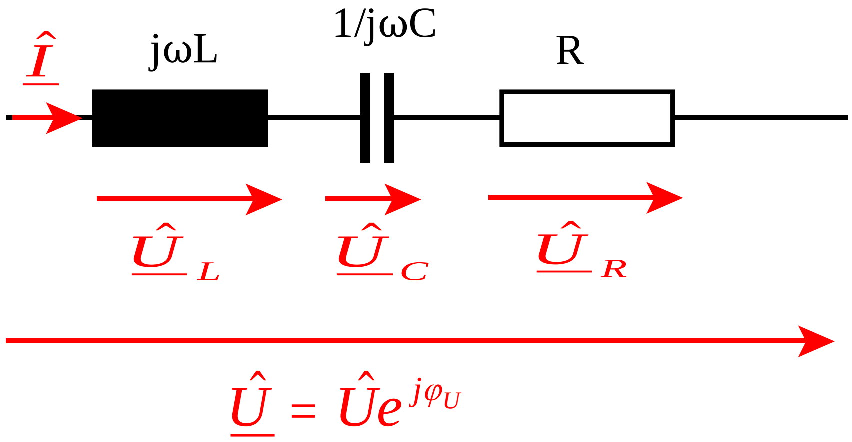
Berechnung der Spannung an der Kapazität in Abhängigkeit von der Frequenz

$$\underline{\hat{U}}_C = \frac{1}{j\omega C}\underline{\hat{I}} = \frac{\omega_0}{\omega}\frac{1}{j\omega_0 C}\underline{\hat{I}}$$

$$\underline{\hat{U}}_C = \frac{-jQ_R\underline{\hat{U}}}{x\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\underline{\hat{U}}_C = \frac{\underline{\hat{U}}}{xj\omega_0 CR\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}} = \frac{-jQ_R}{x\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

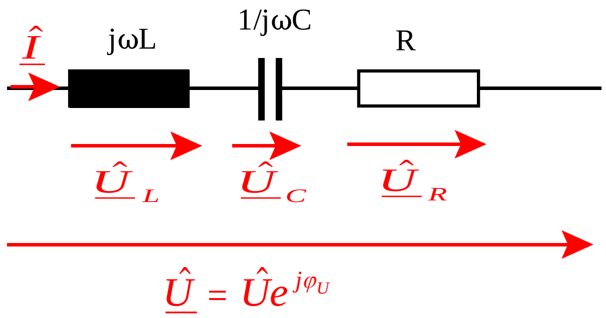


$$\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}} = \frac{-jQ_R}{x\left[1 + jQ_R\left(x - \frac{1}{x}\right)\right]}$$

$$\left|\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}}\right| = \frac{Q_R}{x\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

$$\varphi_{U_C,U} = -\frac{\pi}{2} - \arctan\left(Q_R\left(x - \frac{1}{x}\right)\right)$$

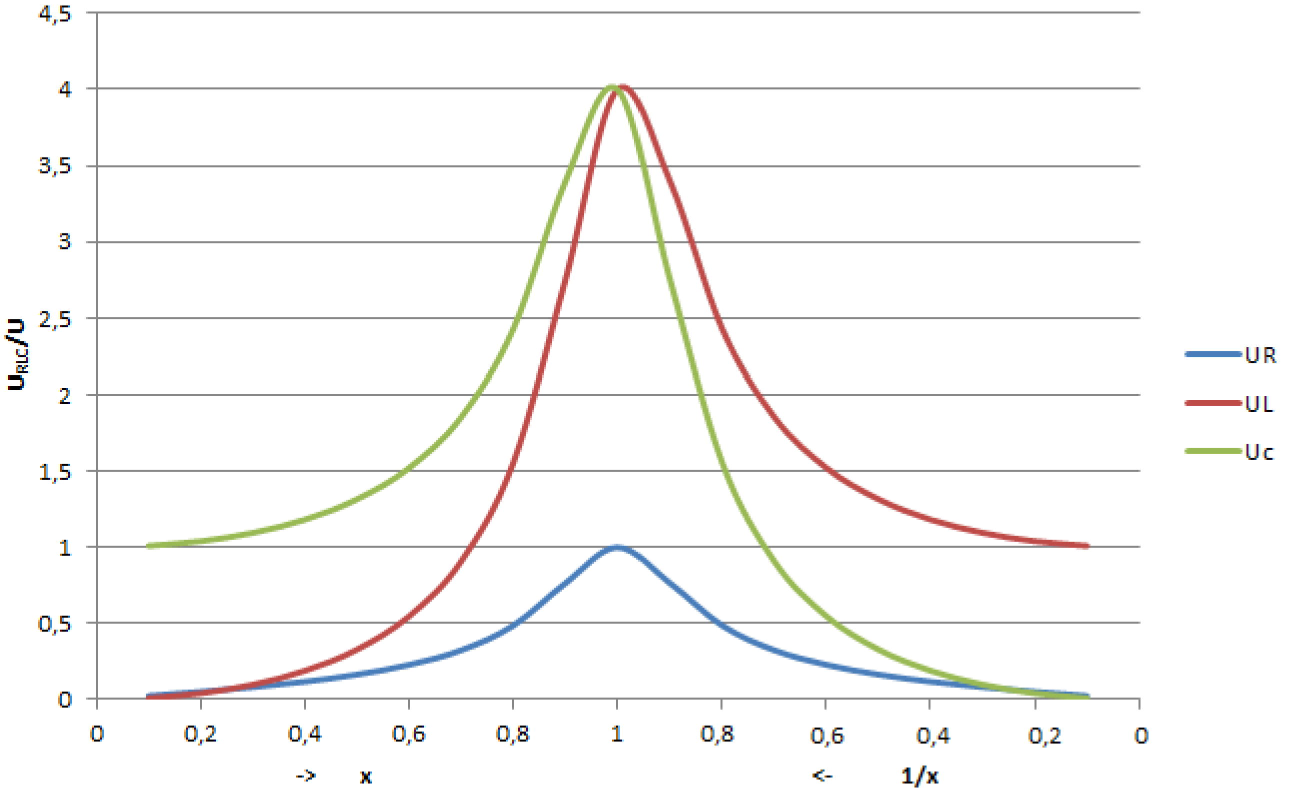
Die Resonanzkurven:



Parameter des Schwingkreises:

L = 0.0001H C = 1.0μF R = 2.5 Ω

$$f_0 = \frac{1}{2\cdot\pi\cdot\sqrt{LC}} = 15{,}9\,\text{kHz} \qquad Q_R = \frac{2\pi f_0 L}{R} = 4$$



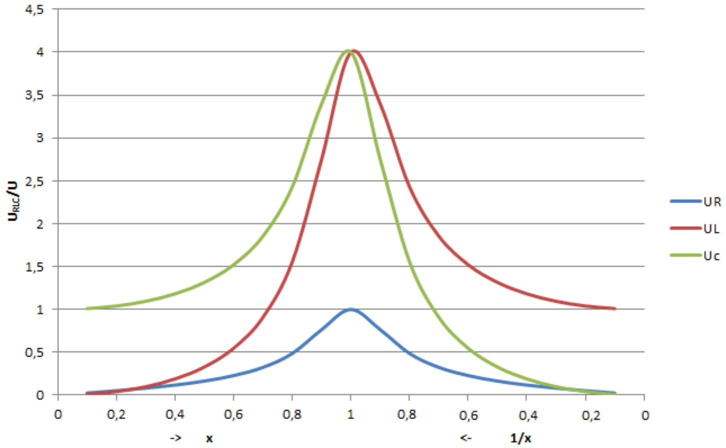
$$\left|\frac{\underline{\hat{U}}_R}{\underline{\hat{U}}}\right| = \frac{1}{\sqrt{1+Q_R^2\left(x-\frac{1}{x}\right)^2}}$$

$$\left|\frac{\underline{\hat{U}}_L}{\underline{\hat{U}}}\right| = \frac{xQ_R}{\sqrt{1+Q_R^2\left(x-\frac{1}{x}\right)^2}}$$

$$\left|\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}}\right| = \frac{Q_R}{x\sqrt{1+Q_R^2\left(x-\frac{1}{x}\right)^2}}$$

Besonderheiten im Resonanzfall:

$$x = 1 \text{ und } x - \frac{1}{x} = 0$$



$$\left|\frac{\hat{U}_R}{\hat{U}}\right| = \frac{1}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

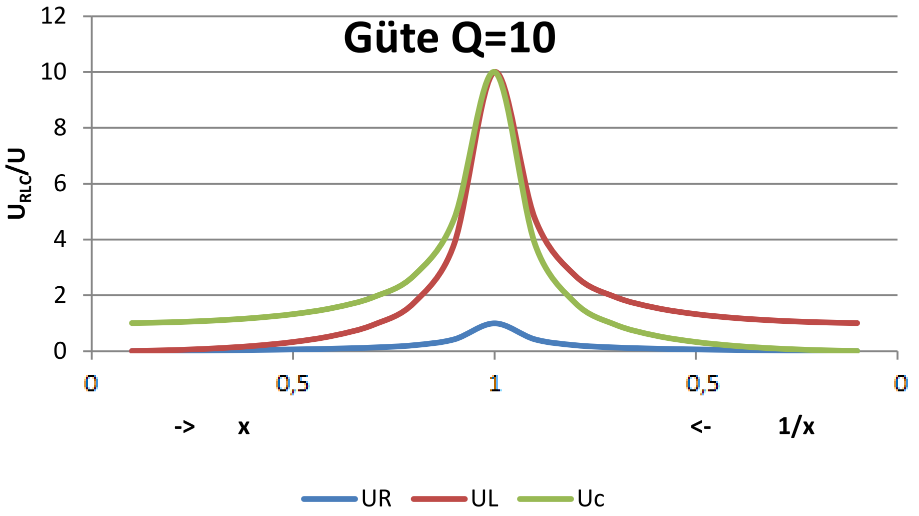
$$\hat{U}_{R0} = \hat{U}$$

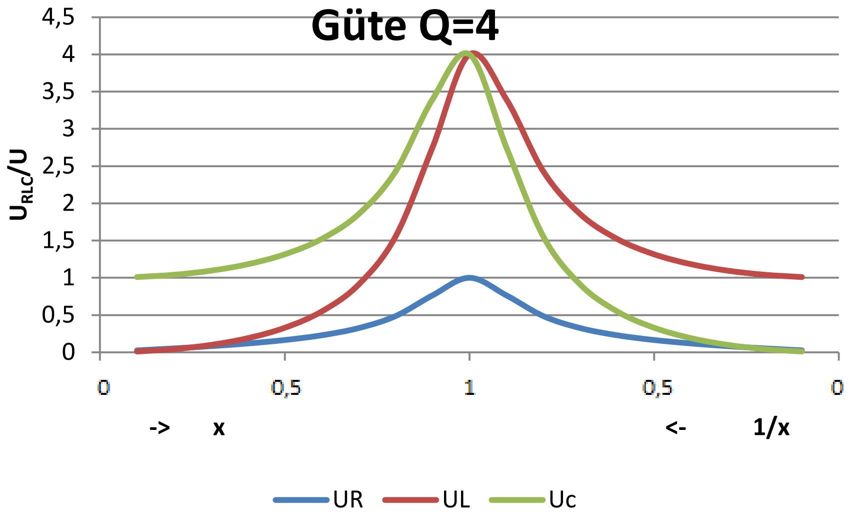
$$\left|\frac{\hat{U}_L}{\hat{U}}\right| = \frac{xQ_R}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

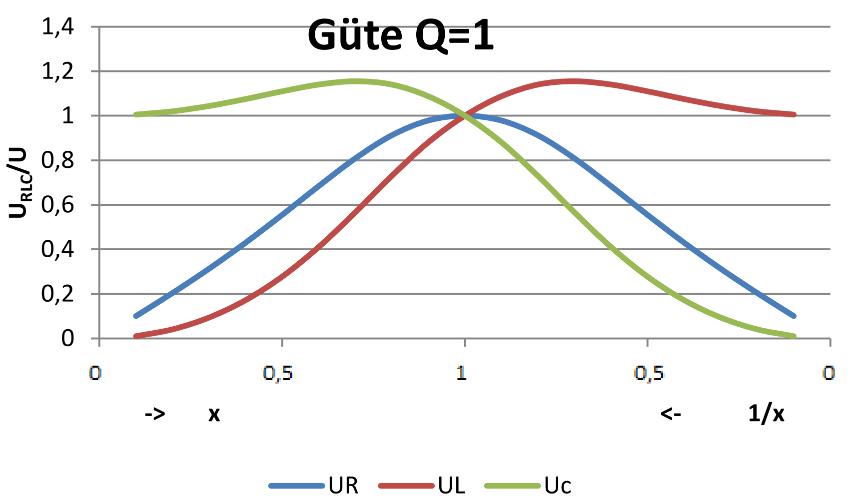
$$\hat{U}_{L0} = Q_R \cdot \hat{U} \text{ (Spannungsüberhöhung)}$$

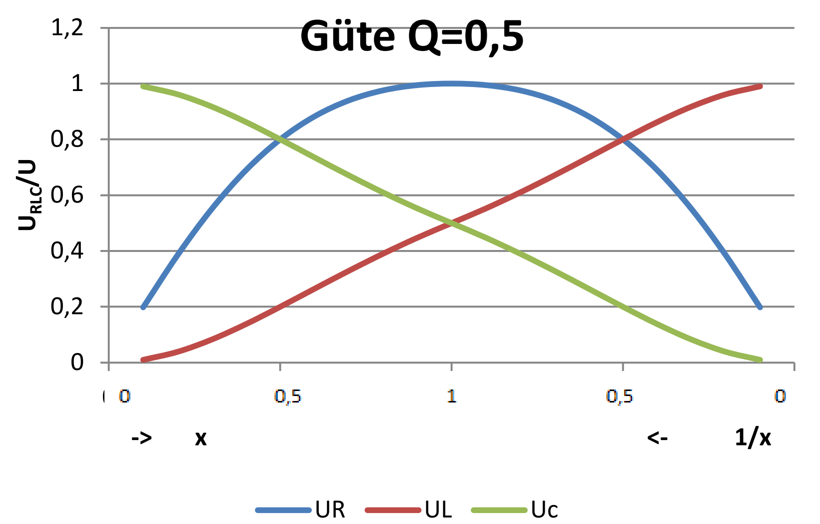
$$\left|\frac{\hat{U}_C}{\hat{U}}\right| = \frac{Q_R}{x\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

$$\hat{U}_{C0} = Q_R \cdot \hat{U} \text{ (Spannungsüberhöhung)}$$

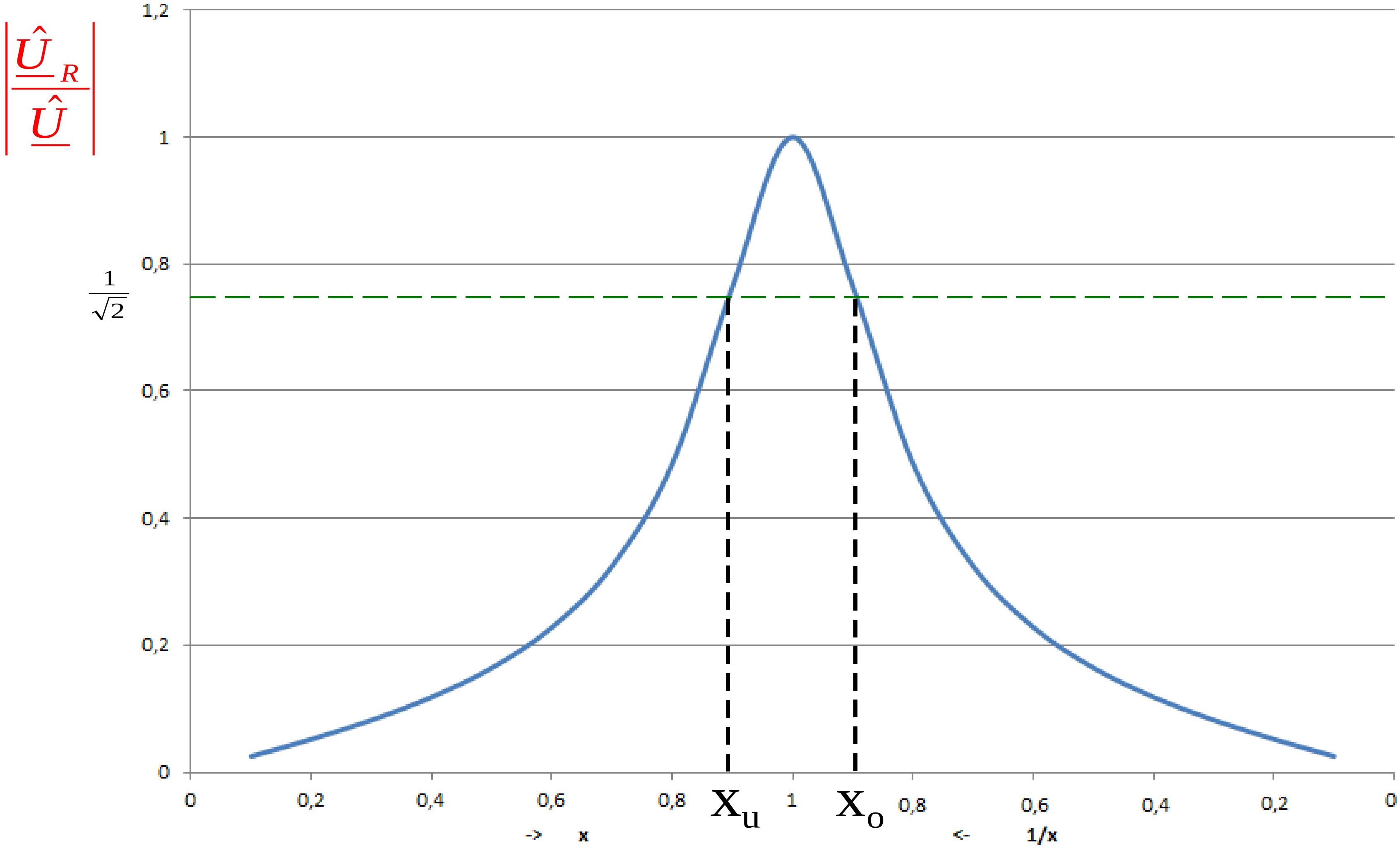

Güte Q=10
12
10
8
6
4
2
0
URLC/U
0
0,5
1
0,5
0
-> x
<- 1/x
UR
UL
Uc

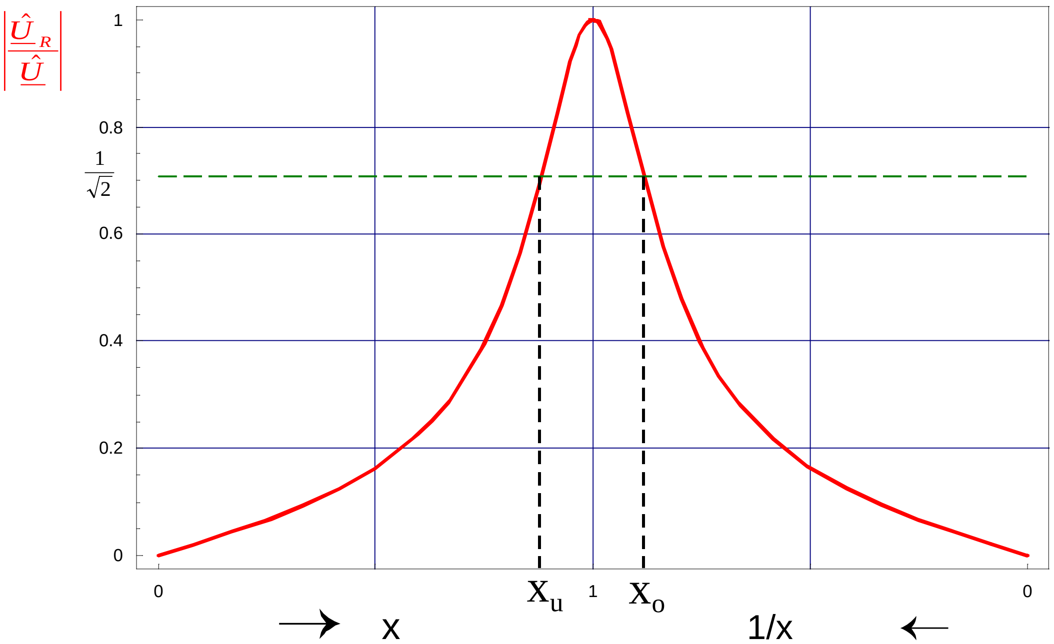

Güte Q=4
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
URLC/U
0
0,5
1
0,5
0
-> x
<- 1/x
UR
UL
Uc


Güte Q=1
1,4
1,2
1
0,8
0,6
0,4
0,2
0
URLC/U
0
0,5
1
0,5
0
-> x
<- 1/x
UR
UL
Uc


Güte Q=0,5
1,2
1
0,8
0,6
0,4
0,2
0
URLC/U
0
0,5
1
0,5
0
-> x
<- 1/x
UR
UL
Uc

- der Zusammenhang zwischen Bandbreite und Güte





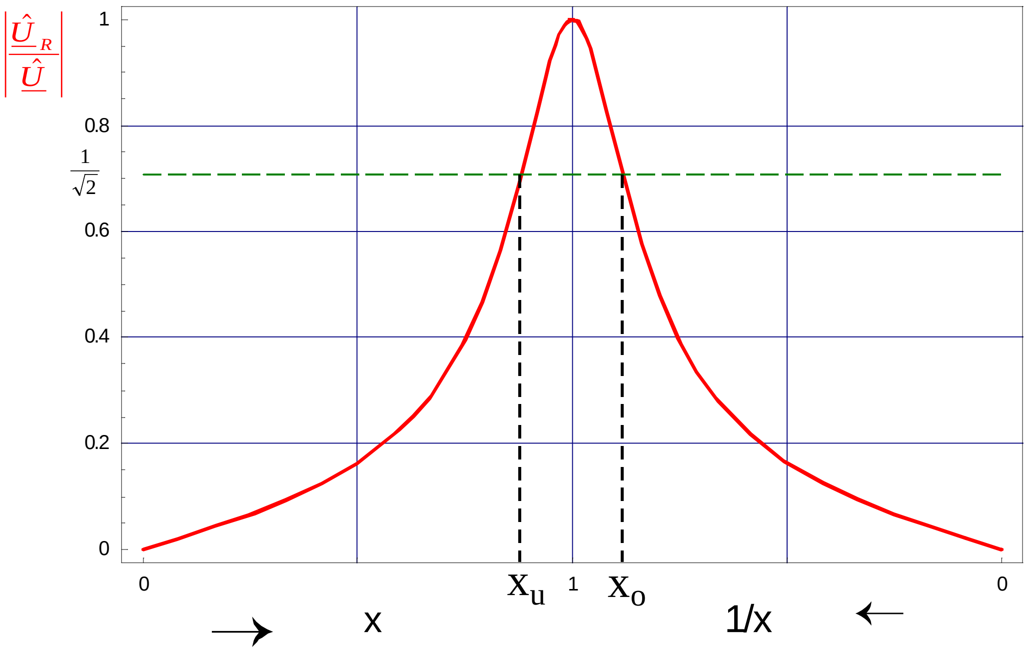
$$\left|\frac{\hat{U}_R}{\hat{U}}\right| = \frac{1}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

f_g oder ω_g:
Im (|H(ω)|=Re(|H(ω)|
(auch 45°-Eckfrequenz genannt)

$$Q_R^2\left(x_{o,u} - \frac{1}{x_{o,u}}\right)^2 = 1$$

$$Q_R\left(x_o - \frac{1}{x_o}\right) = 1 \qquad Q_R\left(x_u - \frac{1}{x_u}\right) = -1 \qquad x_o = \frac{f_{ob}}{f_0} \quad x_u = \frac{f_u}{f_0}$$

$$\frac{f_{ob}}{f_0} - \frac{f_0}{f_{ob}} = \frac{1}{Q_R} \qquad \frac{f_{ob}^2 - f_0^2}{f_0 f_{ob}} = \frac{1}{Q_R} \qquad \frac{\left(f_{ob} + f_0\right)\left(f_{ob} - f_0\right)}{f_0 f_{ob}} = \frac{1}{Q_R}$$



$$\left|\frac{\hat{U}_R}{\hat{U}}\right| = \frac{1}{\sqrt{1 + Q_R^2\left(x - \frac{1}{x}\right)^2}}$$

$$\frac{(f_{ob} + f_0)(f_{ob} - f_0)}{f_0 f_{ob}} = \frac{1}{Q_R}$$

$$x_o = \frac{f_{ob}}{f_0} \qquad x_u = \frac{f_u}{f_0}$$

$$Q_R\left(x_u - \frac{1}{x_u}\right) = -1$$

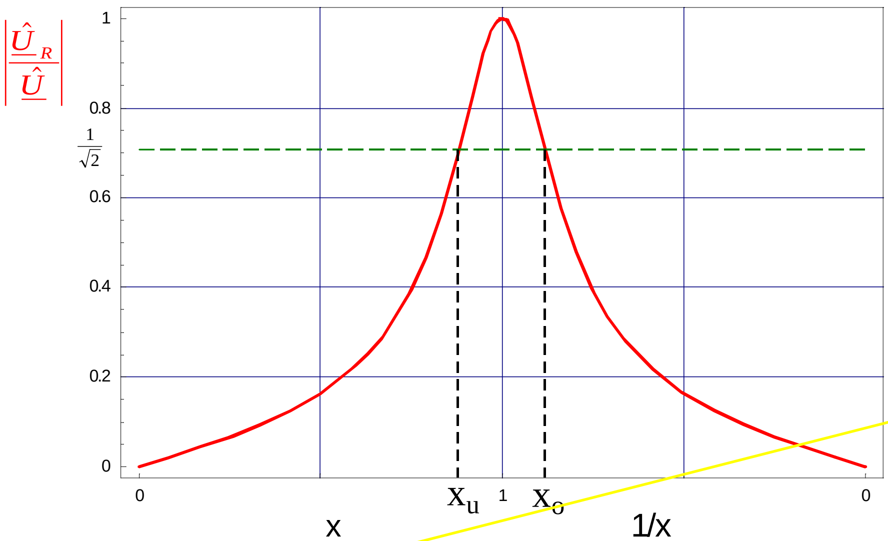
$$\frac{(f_u + f_0)(f_u - f_0)}{f_0 f_u} = -\frac{1}{Q_R}$$

bei großer Güte gilt in guter Näherung

$$f_{ob} \approx f_u \approx f_0$$

und für die Bandbreite b

$$b = f_{ob} - f_u$$



$$\left|\frac{\hat{U}_R}{\hat{U}}\right| = \frac{1}{\sqrt{1+Q_R^2\left(x-\frac{1}{x}\right)^2}}$$

$$f_{ob} \approx f_u \approx f_0$$

$$b = f_{ob} - f_u$$

$$\frac{(f_{ob}+f_0)(f_{ob}-f_0)}{f_0 f_{ob}} = \frac{1}{Q_R}$$

$$\frac{2\cdot f_{ob}\cdot(f_{ob}-f_0)}{f_0\cdot f_{ob}} = \frac{1}{Q_R}$$

$$\frac{2(f_{ob}-f_0)}{f_0} = \frac{1}{Q_R}$$

$$f_{ob} = \left(\frac{1}{2Q_R}+1\right) f_0$$

$$f_u = \left(-\frac{1}{2Q_R}+1\right) f_0$$

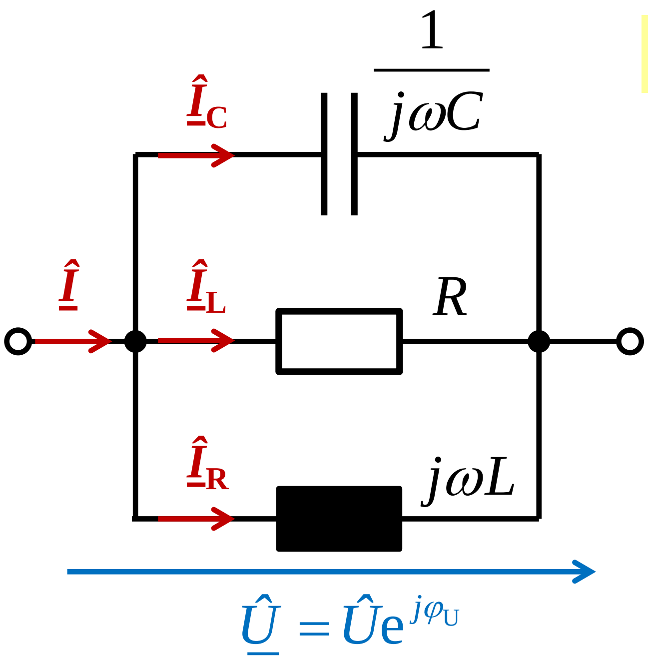
$$b = \left[\frac{1}{2\cdot Q_R}+1-\left(-\frac{1}{2\cdot Q_R}+1\right)\right]\cdot f_0$$

$$\frac{(f_u+f_0)(f_u-f_0)}{f_0 f_u} = -\frac{1}{Q_R}$$

$$\frac{2(f_u-f_0)}{f_0} = -\frac{1}{Q_R}$$

$$b = \frac{1}{Q_R} f_0$$

b) ***die Parallel- oder Stromresonanz*** (der ideale Parallelschwingkreis)



Eingeprägter Strom!

Beispiel:
R=5kΩ
L=1H
C=1µF
Î=10mA

$$\underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

- *Bestimmung der Resonanzfrequenz*

$$\mathrm{Im}\{\underline{Y}(\omega_0)\} = 0$$

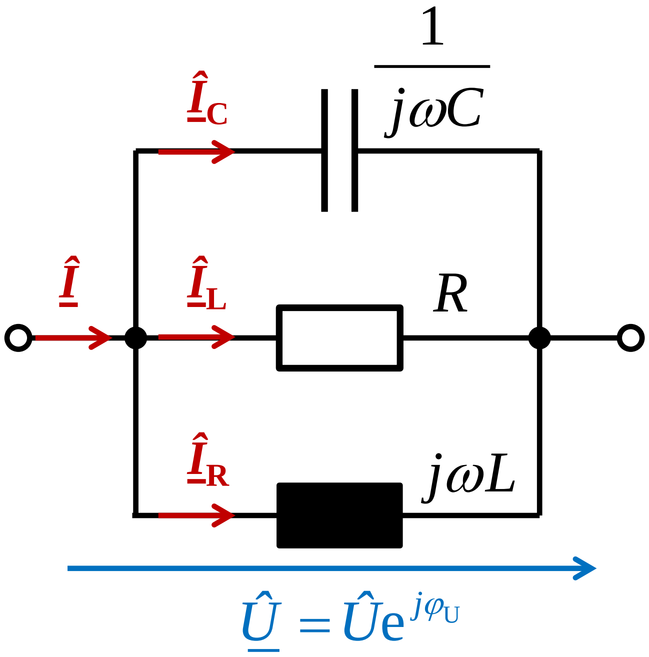
$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonanzfrequenz im Beispielschwingkreis:
f_0=225,08 Hz

- *die Abhängigkeit der Spannung* $\underline{\hat{U}}$ *von der Kreisfrequenz*

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\underline{Y}} = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$



$$\underline{Y} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

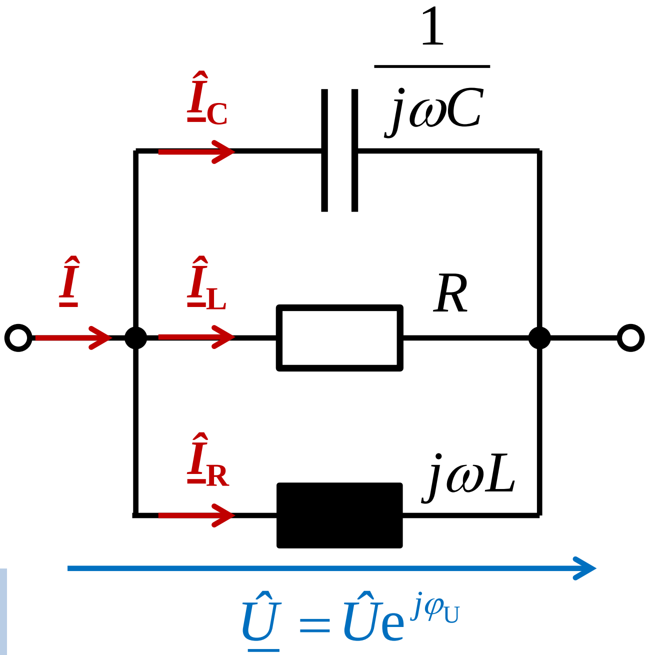
mit der Resonanzkreisfrequenz folgt:

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\left(\frac{\omega}{\omega_0}\omega_0 C - \frac{1}{\frac{\omega}{\omega_0}\omega_0 L}\right)}$$

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\omega_0 C\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)} = \frac{\underline{\hat{I}}}{\frac{1}{R}\left[1 + j\omega_0 CR\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)\right]}$$

$$\underline{\hat{U}} = \frac{\underline{\hat{I}}}{\frac{1}{R}\left[1 + j\omega_0 CR\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)\right]}$$

Mit den Abkürzungen



normierte Frequenz, Güte und Spannung bei Resonanz

$$x = \frac{\omega}{\omega_0} = \frac{f}{f_0}$$

Bei Resonanz gilt:

$$Q_P = \frac{Leistung\ an\ L\ oder\ C}{Leistung\ an\ R}$$

$$Q_P = \frac{U_0^2 / \omega_0 L}{U_0^2 / R} \quad oder \quad \frac{U_0^2 * \omega_0 C}{U_0^2 / R}$$

$$Q_P = \omega_0 CR = \frac{R}{\omega_0 L}$$

Mit den Beispielwerten wird Q_P=3,536

Beispiel:
R=5kΩ
L=1H
C=1µF
Î=10mA

$$\underline{\hat{U}}_0 = \underline{\hat{I}}R$$

Mit den Beispielwerten wird $\hat{U}_0 = 10mA * 5k\Omega = 50V$

$$\underline{\hat{U}} = \frac{\underline{\hat{I}}}{\frac{1}{R}\left[1 + j\omega_0 CR\left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)\right]}$$

$$x = \frac{\omega}{\omega_0} = \frac{f}{f_0} \qquad Q_P = \omega_0 CR = \frac{R}{\omega_0 L} \qquad \underline{\hat{U}}_0 = \underline{\hat{I}}R$$

Aus obigen Beziehungen folgt:

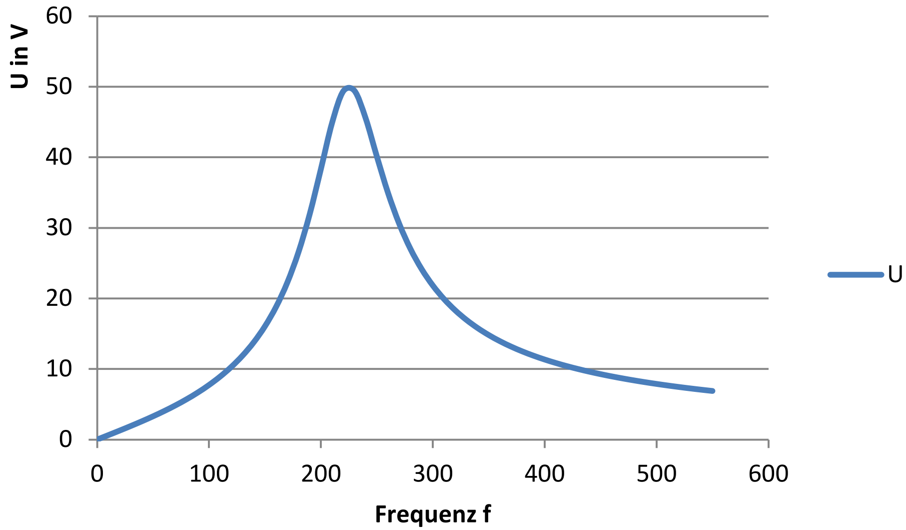
$$\underline{\hat{U}} = \frac{\underline{\hat{U}}_0}{1 + jQ_\mathrm{P}\left(x - \frac{1}{x}\right)} \quad \text{bzw.} \quad \frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_\mathrm{P}\left(x - \frac{1}{x}\right)}$$

mit Betrags- und Phasenverlauf:

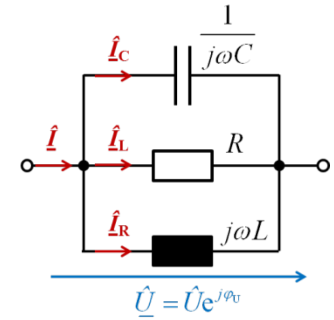
$$\left|\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0}\right| = \frac{1}{\sqrt{1 + Q_\mathrm{P}^2\left(x - \frac{1}{x}\right)^2}}$$

$$\varphi_\mathrm{U} = -\arctan\left(Q_\mathrm{P}\left(x - \frac{1}{x}\right)\right)$$

Mit den Beispielwerten ergibt sich für U:



$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_P\left(x - \frac{1}{x}\right)} \qquad \underline{\hat{U}}_0 = \underline{\hat{I}}R$$



- *die Berechnung des **Stromes durch den Widerstand** in Abhängigkeit von der norm. Frequenz*

$$\underline{\hat{I}}_R = \frac{\underline{\hat{U}}}{R}$$

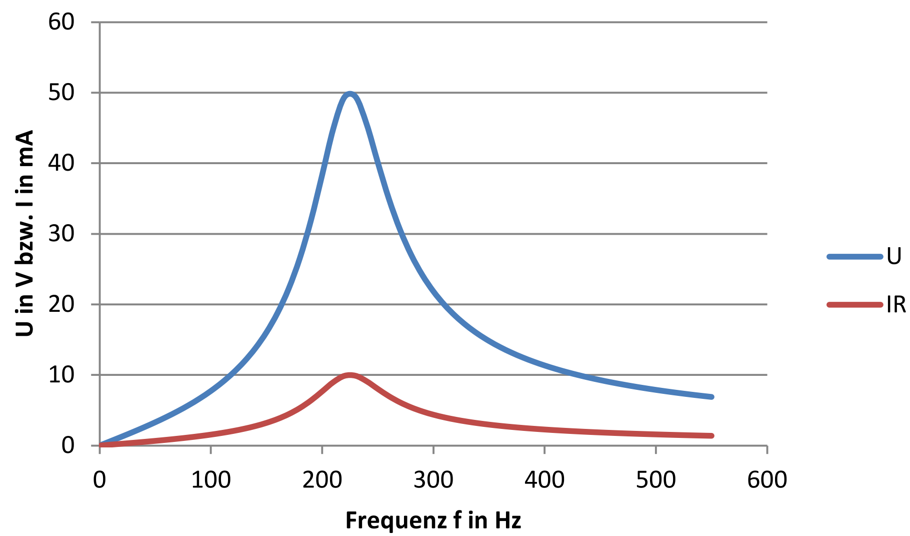
$$\underline{\hat{U}} = \frac{\underline{\hat{I}}}{\frac{1}{R}\left[1 + jQ_P\left(x - \frac{1}{x}\right)\right]}$$

$$\underline{\hat{I}}_R = \frac{1}{R}\frac{\underline{\hat{I}}}{\frac{1}{R}\left[1 + jQ_P\left(x - \frac{1}{x}\right)\right]} = \frac{\underline{\hat{I}}}{1 + jQ_P\left(x - \frac{1}{x}\right)}$$

$$\frac{\underline{\hat{I}}_R}{\underline{\hat{I}}} = \frac{1}{1 + jQ_P\left(x - \frac{1}{x}\right)}$$

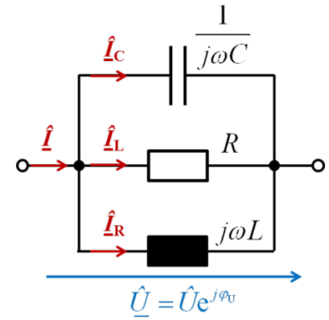
mit Betrags- und Phasenverlauf:

$$\left|\frac{\underline{\hat{I}}_R}{\underline{\hat{I}}}\right| = \frac{1}{\sqrt{1 + Q_P^2\left(x - \frac{1}{x}\right)^2}}$$



$$\varphi_{I_R,I} = -\arctan\left(Q_P\left(x - \frac{1}{x}\right)\right)$$

$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_\mathrm{P}\left(x - \frac{1}{x}\right)} \qquad \underline{\hat{U}}_0 = \underline{\hat{I}}R \qquad Q_P = \omega_0 CR = \frac{R}{\omega_0 L}$$



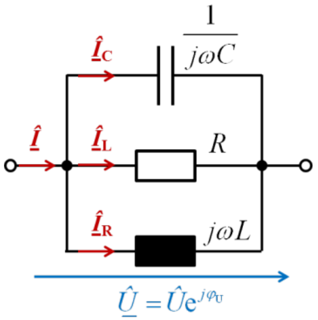
- *die Berechnung des* ***Stromes durch die Induktivität*** *in Abhängigkeit von der norm. Frequenz*

$$\underline{\hat{I}}_\mathrm{L} = \frac{\underline{\hat{U}}}{j\omega L} = \frac{\omega_0}{\omega}\frac{\underline{\hat{U}}}{j\omega_0 L}$$

mit Betrags- und Phasenverlauf:

$$\left|\frac{\underline{\hat{I}}_\mathrm{L}}{\underline{\hat{I}}}\right| = \frac{Q_\mathrm{P}}{x\sqrt{1 + Q_\mathrm{P}^2\left(x - \frac{1}{x}\right)^2}} \qquad \varphi_{I_\mathrm{L},I} = -\frac{\pi}{2} - \arctan\left(Q_\mathrm{P}\left(x - \frac{1}{x}\right)\right)$$

$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_\mathrm{P}\left(x - \frac{1}{x}\right)} \qquad \underline{\hat{U}}_0 = \underline{\hat{I}}R \qquad Q_\mathrm{P} = \omega_0 CR = \frac{\omega_0 L}{R}$$

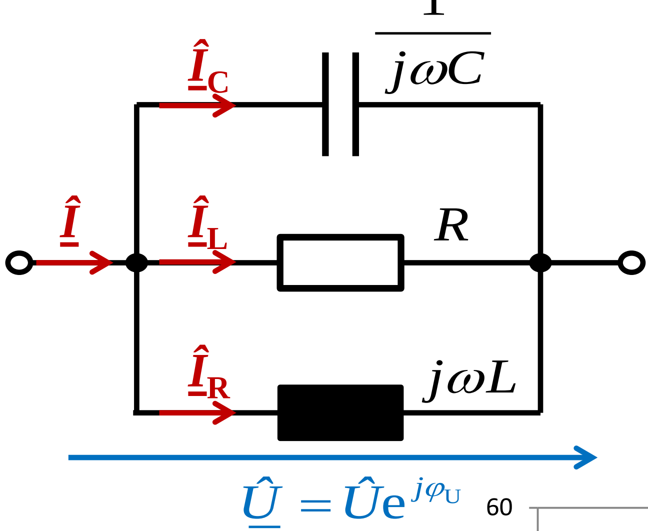


• *die Berechnung des **Stromes durch die Kapazität** in Abhängigkeit von der norm. Frequenz*

$$\underline{\hat{I}}_\mathrm{C} = j\omega C\underline{\hat{U}} = \frac{\omega}{\omega_0} j\omega_0 C\underline{\hat{U}}$$

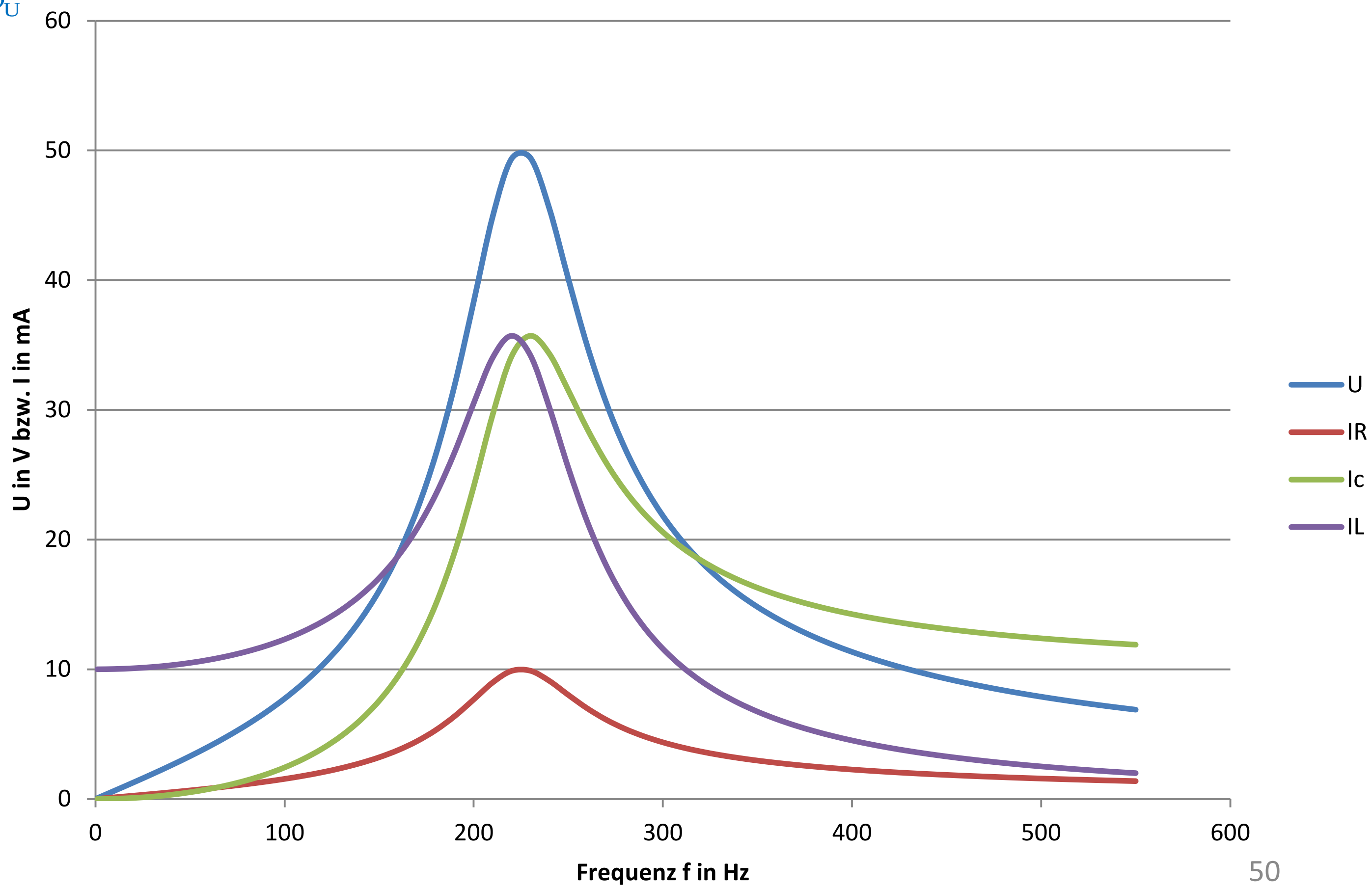
mit Betrags- und Phasenverlauf:

$$\left|\frac{\underline{\hat{I}}_\mathrm{C}}{\underline{\hat{I}}}\right| = \frac{xQ_\mathrm{P}}{\sqrt{1 + Q_\mathrm{P}^2\left(x - \frac{1}{x}\right)^2}} \qquad \varphi_{I_\mathrm{C},I} = \frac{\pi}{2} - \arctan\left(Q_\mathrm{P}\left(x - \frac{1}{x}\right)\right)$$

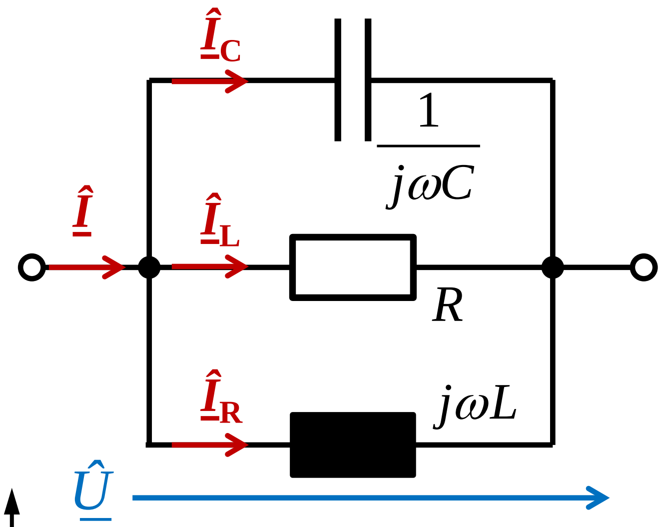


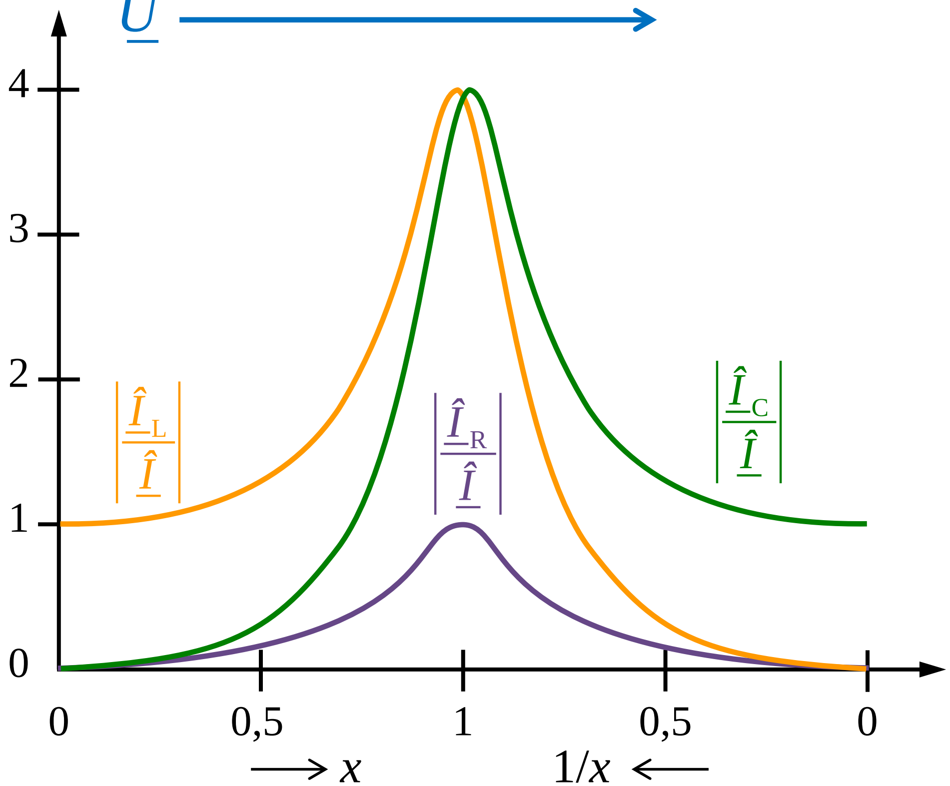
Beispiel:
R=5kΩ
L=1H
C=1µF
Î=10mA

f_0=225,08 Hz
Q_P =3,536
$\hat{I}_{L0}=\hat{I}_{C0}$=35,36 mA



- *die Resonanzkurven*





Parameter des Schwingkreises:

$L = 0{,}1\ \text{mH} \qquad C = 1\ \mu\text{F} \qquad R = 2{,}5\ \Omega$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \approx 15{,}9\ \text{kHz}$$

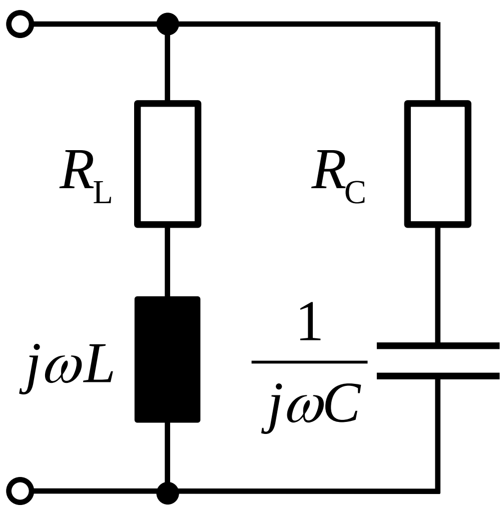
$$Q_P = \frac{2\pi f_0 C}{R} = 4$$

$$\left|\frac{\underline{\hat{I}}_R}{\underline{\hat{I}}}\right| = \frac{1}{\sqrt{1 + Q_P^2\left(x - \frac{1}{x}\right)^2}}$$

$$\left|\frac{\underline{\hat{I}}_C}{\underline{\hat{I}}}\right| = \frac{xQ_P}{\sqrt{1 + Q_P^2\left(x - \frac{1}{x}\right)^2}}$$

$$\left|\frac{\underline{\hat{I}}_L}{\underline{\hat{I}}}\right| = \frac{Q_P}{x\sqrt{1 + Q_P^2\left(x - \frac{1}{x}\right)^2}}$$

b) *der reale Parallelschwingkreis*



$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{1}{j\omega C}}$$

$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{j\omega C}{j\omega C R_C + 1}$$

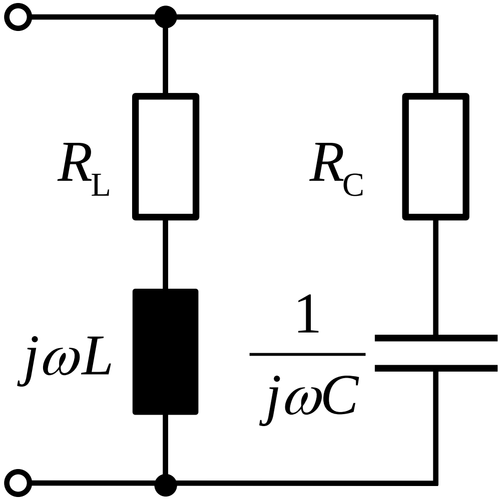
$$\underline{Y} = \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{j\omega C\left(-j\omega C R_C + 1\right)}{(\omega C R_C)^2 + 1}$$

$$\underline{Y} = \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{(\omega C)^2 R_C + j\omega C}{(\omega C R_C)^2 + 1}$$

- *Bestimmung der Resonanzfrequenz*

$$\mathrm{Im}\{\underline{Y}(\omega_0)\} = 0$$

$$\mathrm{Im}\{\underline{Y}\} = \frac{-\omega L}{R_L^2 + (\omega L)^2} + \frac{\omega C}{(\omega C R_C)^2 + 1}$$



$$\frac{-\omega_0 L}{R_L^2+(\omega_0 L)^2}+\frac{\omega_0 C}{(\omega_0 C R_C)^2+1}=0$$

$$\frac{\omega_0 C}{(\omega_0 C R_C)^2+1}=\frac{\omega_0 L}{R_L^2+(\omega_0 L)^2}$$

$$C\left(R_L^2+(\omega_0 L)^2\right)=L\left((\omega_0 C R_C)^2+1\right)$$

$$CR_L^2+\omega_0^2 L^2 C=\omega_0^2 C^2 L R_C^2+L$$

$$\omega_0^2(L^2C-C^2LR_C^2)=L-CR_L^2$$

$$\omega_0^2=\frac{L-CR_L^2}{L^2C-C^2LR_C^2}$$

$$\omega_0=\sqrt{\frac{L-CR_L^2}{L^2C-C^2LR_C^2}}$$

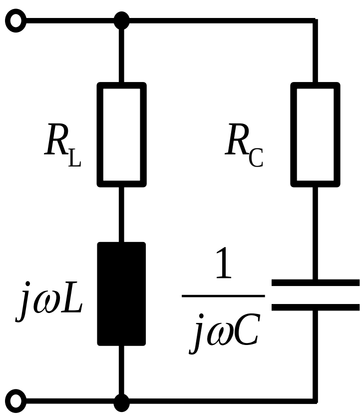
$$\boxed{\omega_0=\frac{1}{\sqrt{LC}}\sqrt{\frac{\frac{L}{C}-R_L^2}{\frac{L}{C}-R_C^2}}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}} \qquad \omega_0 = \sqrt{\frac{L - CR_L^2}{L^2C - C^2LR_C^2}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{L - CR_L^2}{L - CR_C^2}}$$

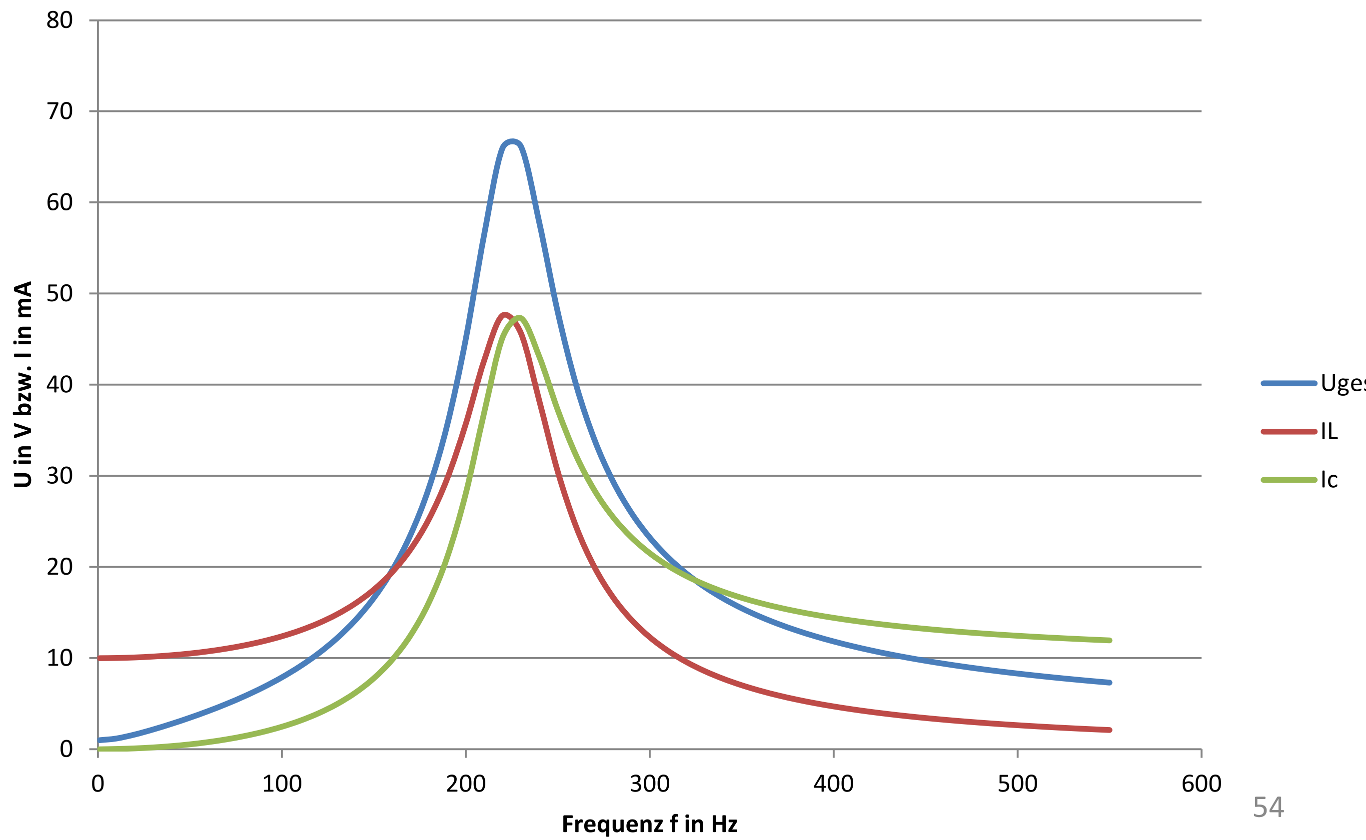
- *Untersuchungen zum Schwingverhalten*

1. Spezialfall: $\frac{L}{C} > R_L^2 \text{ und } \frac{L}{C} > R_C^2$ oder $\frac{L}{C} < R_L^2 \text{ und } \frac{L}{C} < R_C^2$

ω_0 ist reell → Resonanz = schwache Dämpfung (Schwingfall)



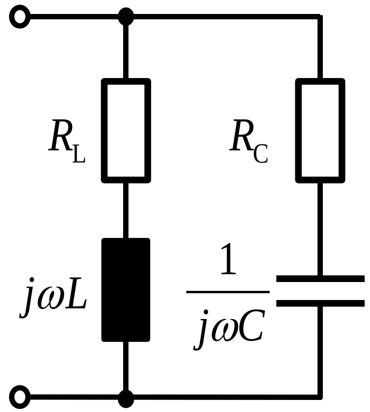
L=1H, C=0,5µF
R_L=100Ω, R_C=200Ω
L/C=2000000
I_{ges}=10mA
f_0=226,8Hz
f_{0*} (ohne R_L und R_C =225,08Hz)

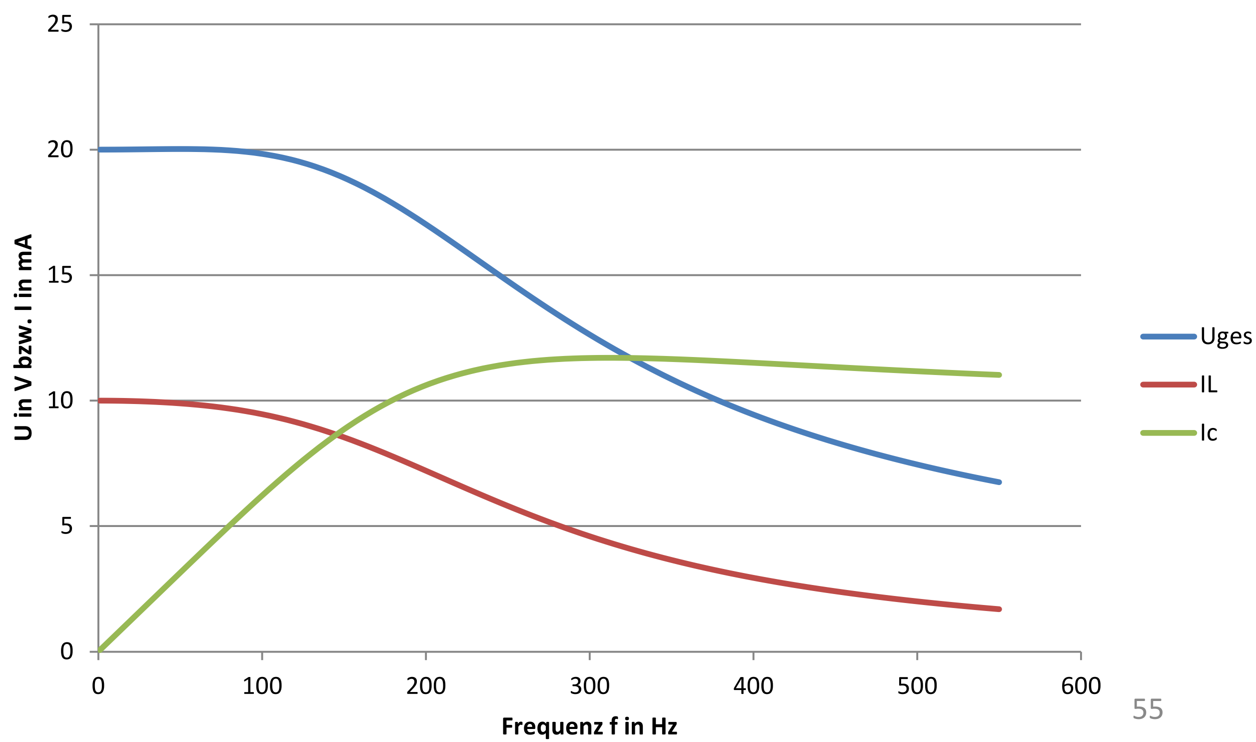


$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}}$$

2. Spezialfall: $\frac{L}{C} < R_L^2 \text{ und } \frac{L}{C} > R_C^2$ oder $\frac{L}{C} > R_L^2 \text{ und } \frac{L}{C} < R_C^2$

$\boldsymbol{\omega_0}$ ist imaginär → keine Resonanz = überkritische Dämpfung (Kriechfall)





L=1H, C=0,5µF
R_L=2000Ω,
R_C=200Ω
L/C=2000000
I_{ges}=10mA

Keine reelle Lösung für f_0

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_\mathrm{L}^2}{\frac{L}{C} - R_\mathrm{C}^2}}$$

---

3. Spezialfall: $\frac{L}{C} = R_L^2 \; und \; \frac{L}{C} = R_C^2$

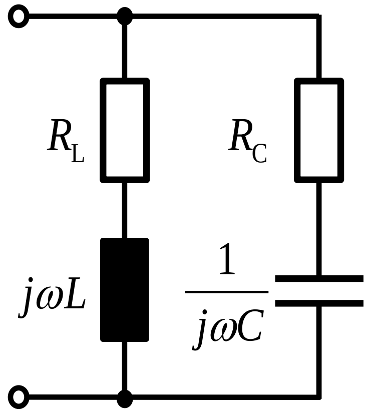
$$\mathrm{Im}\{\underline{Y}\} = \frac{-\omega L}{R_\mathrm{L}^2 + (\omega L)^2} + \frac{\omega C}{(\omega C R_\mathrm{C})^2 + 1}$$

$$\mathrm{Im}\{\underline{Y}\} = \frac{-\omega L}{\frac{L}{C} + (\omega L)^2} + \frac{\omega C}{(\omega C)^2 \frac{L}{C} + 1}$$

$$\mathrm{Im}\{\underline{Y}\} = \frac{-\omega}{\frac{1}{C} + \omega^2 L} + \frac{\omega C}{\omega^2 CL + 1}$$

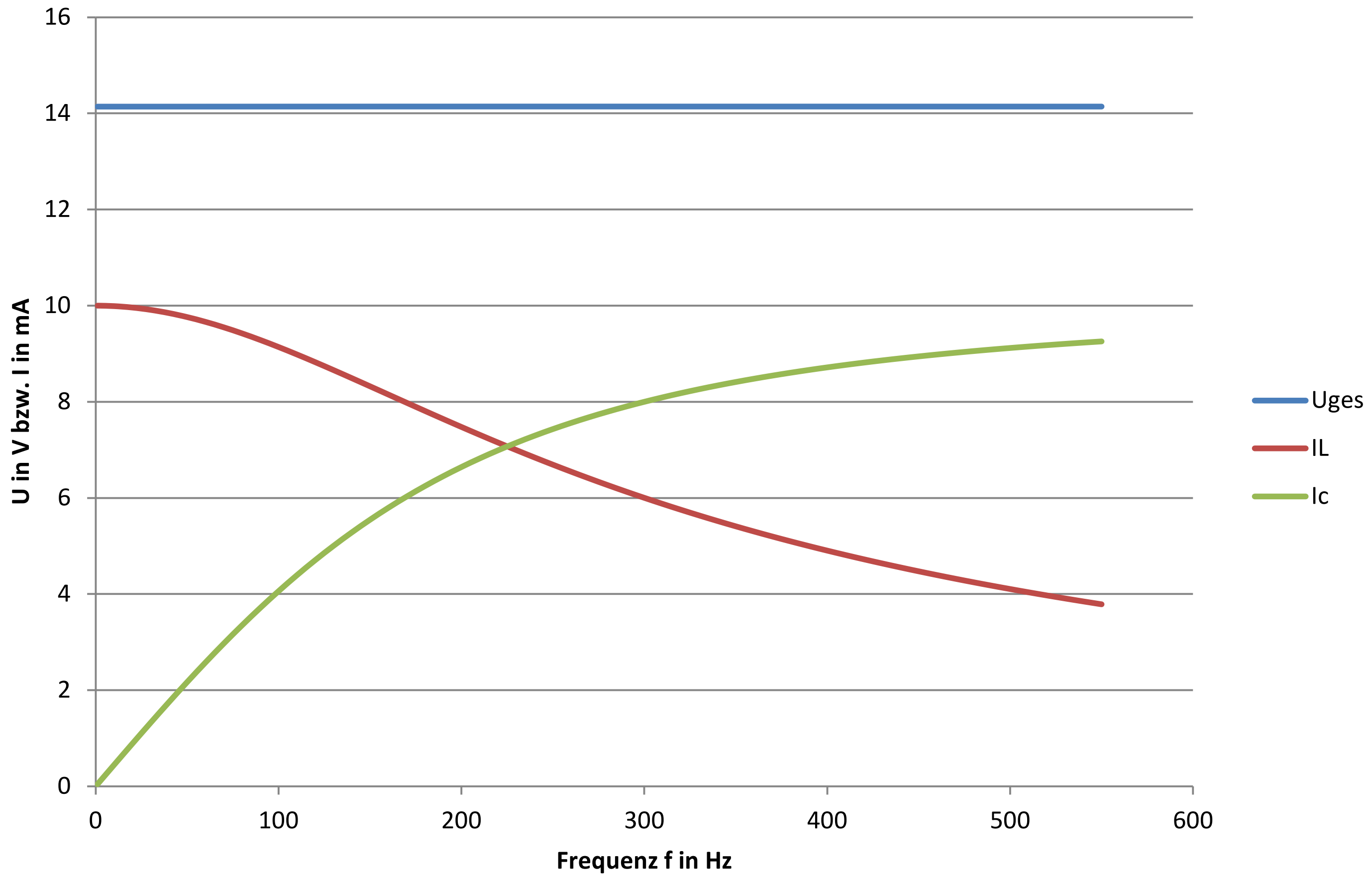
$$\mathrm{Im}\{\underline{Y}\} = \frac{-\omega C}{1 + \omega^2 LC} + \frac{\omega C}{\omega^2 CL + 1} = 0$$

Imaginärteil verschwindet für alle $\boldsymbol{\omega}$ → ewige Resonanz

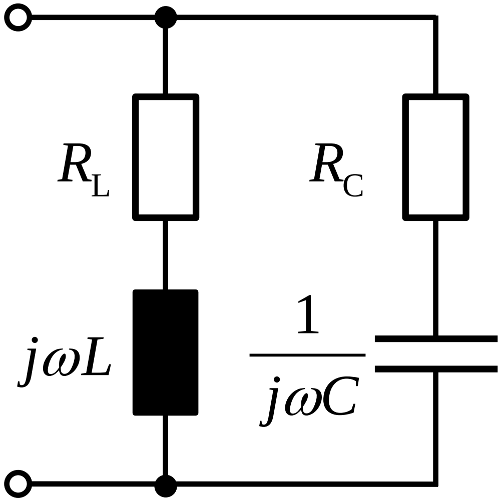


L=1H, C=0,5µF
R_L=1414Ω,
R_C=1414Ω
L/C=2000000
I_{ges}=10mA

Keine reelle Lösung für f_0



- *der Resonanzwiderstand bei* $\boldsymbol{R_\mathrm{C} = 0}$



$$\underline{Y} = \frac{1}{R_\mathrm{L} + j\omega L} + \frac{1}{R_\mathrm{C} + \frac{1}{j\omega C}}$$

$R_\mathrm{C} = 0$

$$\underline{Y} = \frac{1}{R_\mathrm{L} + j\omega L} + j\omega C = \frac{R_\mathrm{L} - j\omega L}{R_\mathrm{L}^2 + (\omega L)^2} + j\omega C$$

$$\underline{Y}(\omega_0) = \frac{R_\mathrm{L}}{R_\mathrm{L}^2 + (\omega_0 L)^2}$$

$$\omega_0^2 = \frac{L - CR_\mathrm{L}^2}{L^2C - C^2LR_\mathrm{C}^2}$$

$R_\mathrm{C} = 0$

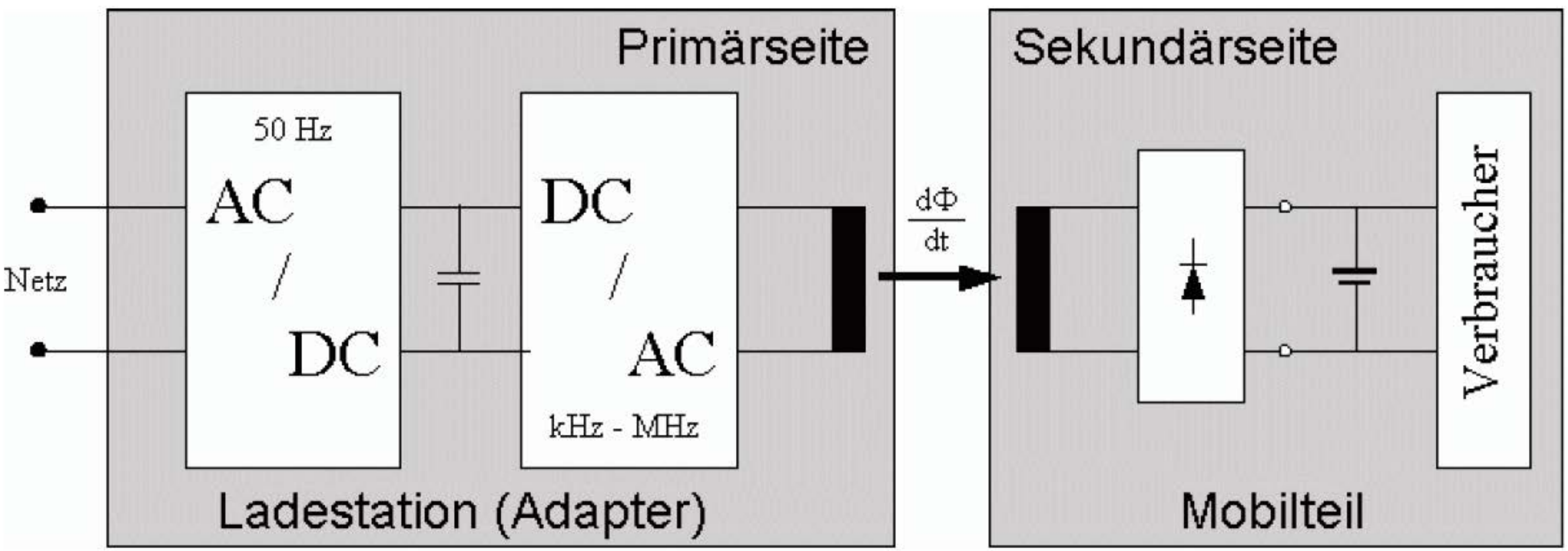
$$\omega_0^2 = \frac{L - CR_\mathrm{L}^2}{L^2C}$$

$$\underline{Y}(\omega_0) = \frac{R_\mathrm{L}}{R_\mathrm{L}^2 + \frac{L - CR_\mathrm{L}^2}{L^2C} L^2} = \frac{R_\mathrm{L}}{R_\mathrm{L}^2 + \frac{L - CR_\mathrm{L}^2}{C}}$$

$$\underline{Y}(\omega_0) = \frac{R_\mathrm{L}}{R_\mathrm{L}^2 + \frac{L}{C} - R_\mathrm{L}^2}$$

$$\underline{Y}(\omega_0) = \frac{R_\mathrm{L}}{\frac{L}{C}}$$ bzw. $$\underline{Z}(\omega_0) = \frac{1}{\underline{Y}(\omega_0)} = \frac{L}{CR_\mathrm{L}}$$

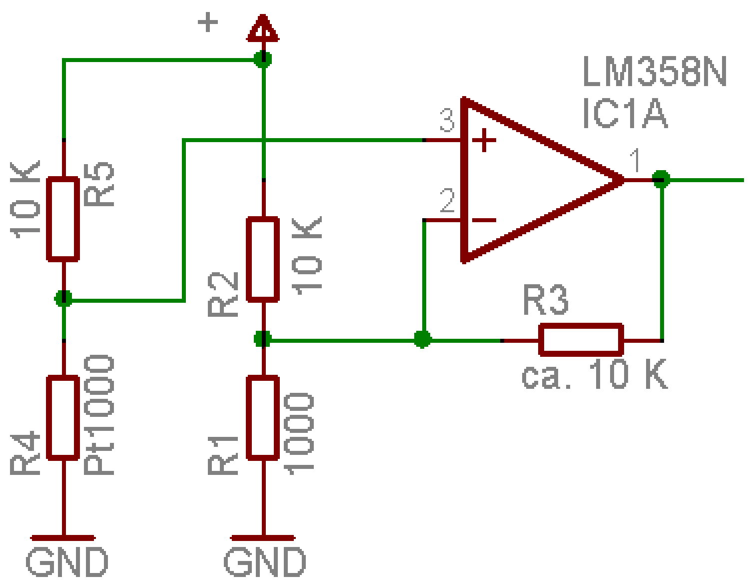
Blockschaltbild eines induktiven Ladegerätes



Induktive Ladestation

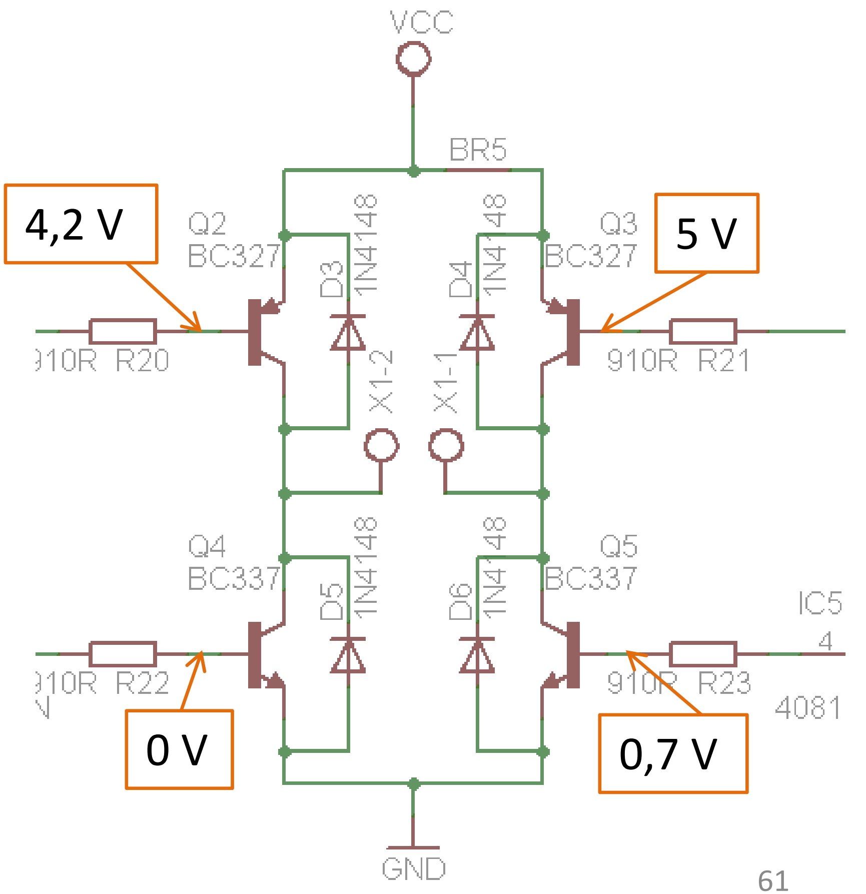
Anwendung Brücken: Sensortechnik

(Temperaturmessung mit Pt1000)



Anwendung Brücken: Motorsteuerung

(Potentiale für Rechtslauf)



Vertiefung und Visualisierung:

GETsoft

LearnWeb | TaskWeb | LabWeb | TestWeb | BookWeb | über GETsoft

... stellt Elektrotechnik-Lernprogramme mit Kompendien, Beispiel- und Übungsaufgaben mit abgestufter Hilfe, Experimentierumgebungen, Glossarien u.v.m. zur Verfügung

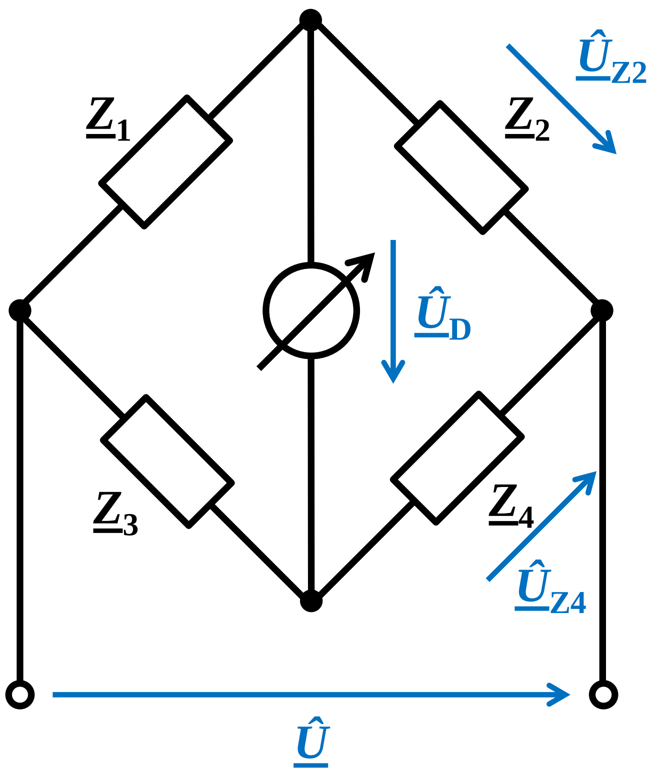
GETsoft » LearnWeb » Messbrücken

- Brückenkurs GET - Mathematik
- Grundstromkreis
- Gleichstromnetze
- Ausgleichsvorgänge
- Induktionsvorgänge
- Frequenzselektive Schaltungen
- Messbrücken
- Transformator
- Drehstromsystem
- Fourier-Reihen
- Fourier-Transformation
- Laplace-Transformation
- Leitungen

Messbrücken

Konzept: Das Lernprogramm liefert ein Angebot von strukturiertem Kernwissen über Gleichstrom- und Wechselstrommessbrücken anhand:

- der Analyse der Brückenschaltung
- der Herleitung der Abgleichbedingungen bekannter Brückenanordnungen wie z.B. der Resonanzmessbrücke, der induktiven und kapazitiven Vergleichsbrücke, der Induktivitätsmessbrücke nach Maxwell-Wien u.a.
- von Aussagen zur Konvergenz des Abgleichs
- des Vergleichs der Anordnungen
- von Visualisierung durch Java-Applets



Abgleich:

$$\underline{\hat{U}}_{\mathrm{D}} = 0$$

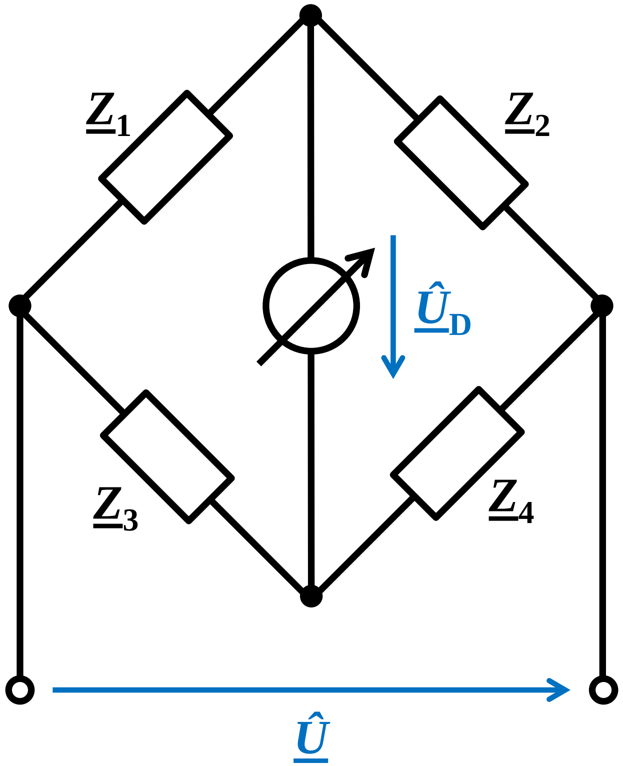
$$\underline{\hat{I}}_{\mathrm{D}} = 0$$

$$\underline{\hat{U}}_{\mathrm{Z2}} = \underline{\hat{U}}_{\mathrm{Z4}}$$

$$\underline{\hat{U}}_{\mathrm{Z2}} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{\hat{U}}$$

$$\underline{\hat{U}}_{\mathrm{Z4}} = \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4}\underline{\hat{U}}$$

$$\underline{\hat{U}}_{Z2} = \underline{\hat{U}}_{Z4} \qquad \underline{\hat{U}}_{Z2} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{\hat{U}} \qquad \underline{\hat{U}}_{Z4} = \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4}\underline{\hat{U}}$$



$$\frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{\hat{U}} = \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4}\underline{\hat{U}}$$

$$\underline{Z}_2(\underline{Z}_3 + \underline{Z}_4) = \underline{Z}_4(\underline{Z}_1 + \underline{Z}_2)$$

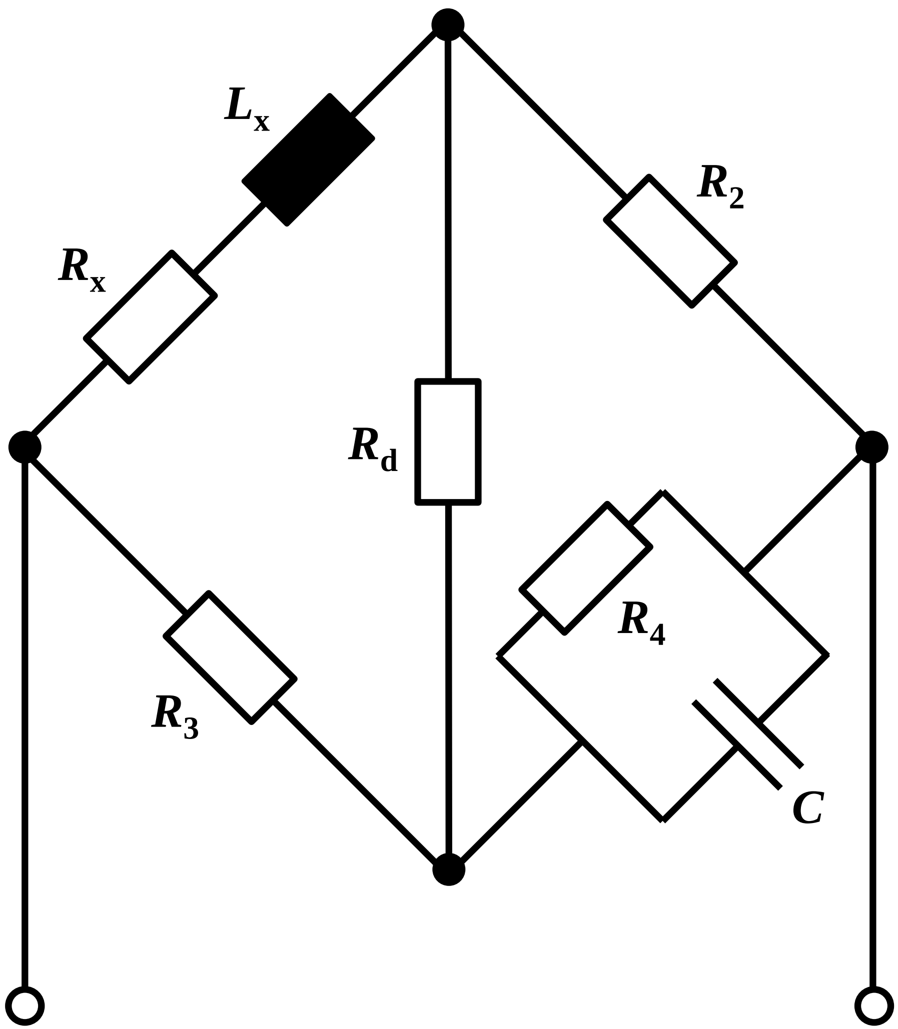
$$\underline{Z}_2\underline{Z}_3 + \underline{Z}_2\underline{Z}_4 = \underline{Z}_1\underline{Z}_4 + \underline{Z}_2\underline{Z}_4$$

$$\underline{Z}_2\underline{Z}_3 = \underline{Z}_1\underline{Z}_4$$

Abgleichbedingung:

$$\boxed{\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}}$$

7.2.1 Die Induktivitätsmessbrücke nach Maxwell-Wien



Abgleichbedingung:

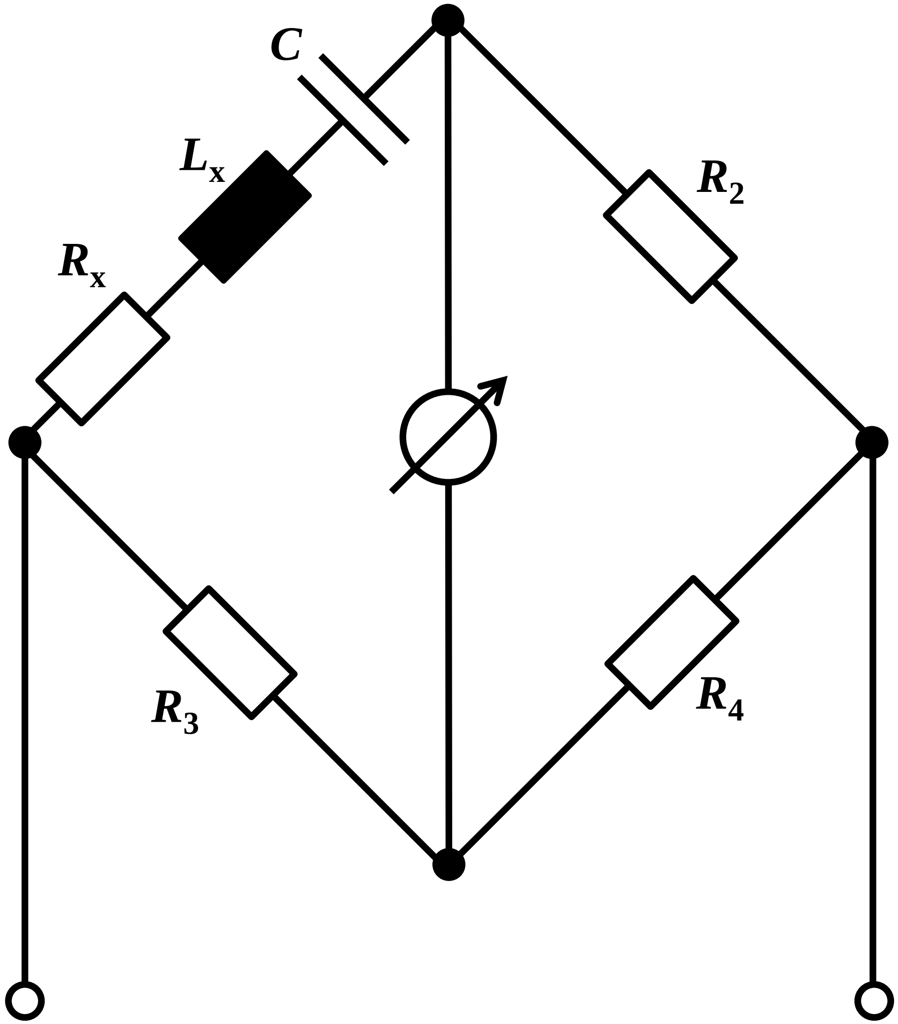
$$\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}$$

$$\frac{R_x + j\omega L_x}{R_2} = R_3\left(\frac{1}{R_4} + j\omega C\right)$$

$$\frac{R_x}{R_2} = \frac{R_3}{R_4} \qquad \frac{\omega L_x}{R_2} = \omega C R_3$$

$$R_x = \frac{R_3}{R_4} R_2 \qquad L_x = C R_2 R_3$$

7.2.2 Die Resonanzmessbrücke



Abgleichbedingung:

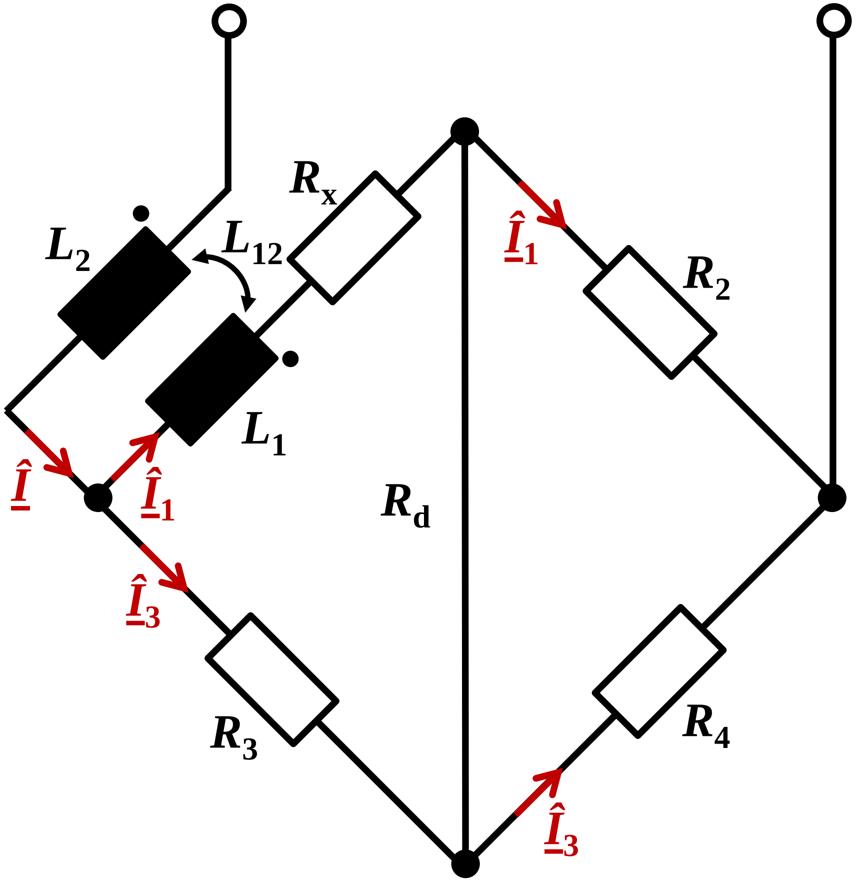
$$\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}$$

$$\frac{R_\mathrm{x} + j\omega L_\mathrm{x} + \dfrac{1}{j\omega C}}{R_2} = \frac{R_3}{R_4}$$

$$\frac{R_\mathrm{x}}{R_2} = \frac{R_3}{R_4} \qquad \omega L_\mathrm{x} - \frac{1}{\omega C} = 0$$

$$R_\mathrm{x} = \frac{R_2 R_3}{R_4} \qquad L_\mathrm{x} = \frac{1}{\omega^2 C}$$

7.2.3 Die Gegeninduktivitätsmessbrücke nach Wien



Abgleichbedingung:

$$\hat{\underline{I}}_D = 0$$

$$\hat{\underline{U}}_1 = \hat{\underline{U}}_3 \qquad \hat{\underline{U}}_2 = \hat{\underline{U}}_4$$

$$\hat{\underline{U}}_1 = \left(R_1 + j\omega L_1\right)\hat{\underline{I}}_1 - j\omega L_{12}\hat{\underline{I}}$$

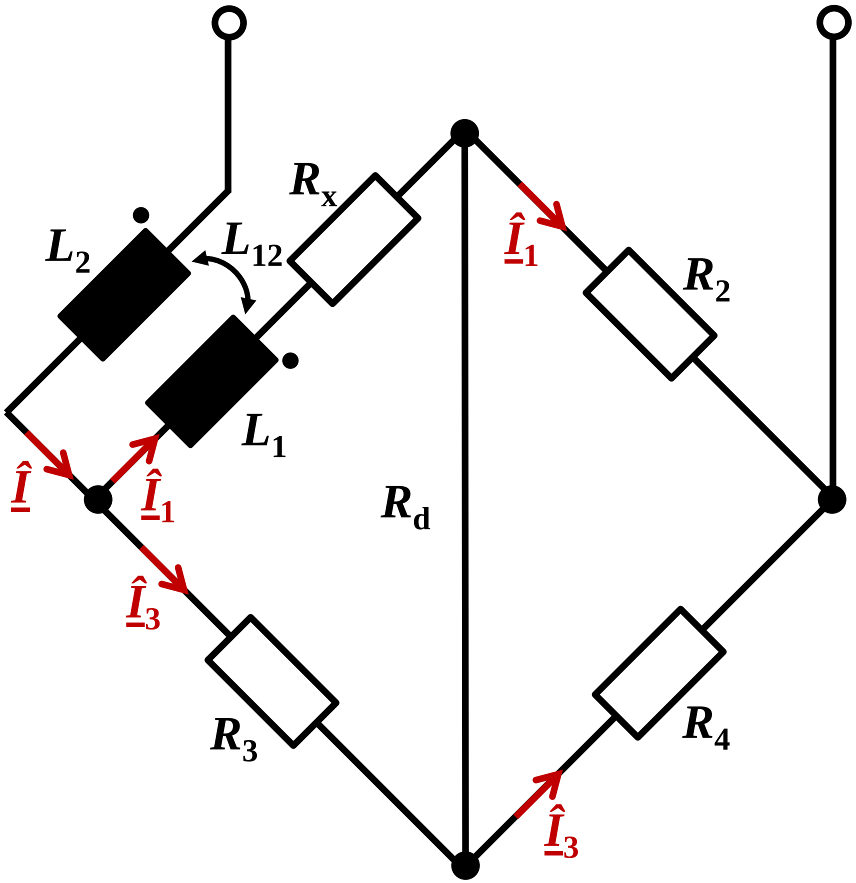
$$\hat{\underline{U}}_3 = R_3\hat{\underline{I}}_3 \qquad \hat{\underline{I}} = \hat{\underline{I}}_1 + \hat{\underline{I}}_3$$

$$\hat{\underline{U}}_2 = R_2\hat{\underline{I}}_1 \qquad \hat{\underline{U}}_4 = R_4\hat{\underline{I}}_3$$

$$\left(R_1 + j\omega L_1\right)\hat{\underline{I}}_1 - j\omega L_{12}\left(\hat{\underline{I}}_1 + \hat{\underline{I}}_3\right) = R_3\hat{\underline{I}}_3$$

$$\left(R_1 + j\omega L_1 - j\omega L_{12}\right)\hat{\underline{I}}_1 = \left(R_3 + j\omega L_{12}\right)\hat{\underline{I}}_3$$

$$R_2\hat{\underline{I}}_1 = R_4\hat{\underline{I}}_3$$



$$\left(R_1 + j\omega L_1 - j\omega L_{12}\right)\underline{\hat{I}}_1 = \left(R_3 + j\omega L_{12}\right)\underline{\hat{I}}_3$$

$$R_2\underline{\hat{I}}_1 = R_4\underline{\hat{I}}_3$$

$$\frac{\left(R_1 + j\omega L_1 - j\omega L_{12}\right)\underline{\hat{I}}_1}{R_2\underline{\hat{I}}_1} = \frac{\left(R_3 + j\omega L_{12}\right)\underline{\hat{I}}_3}{R_4\underline{\hat{I}}_3}$$

$$\frac{\left(R_1 + j\omega L_1 - j\omega L_{12}\right)}{R_2} = \frac{\left(R_3 + j\omega L_{12}\right)}{R_4}$$

$$\frac{L_1 - L_{12}}{R_2} = \frac{L_{12}}{R_4}$$

$$\frac{L_1}{R_2} = \frac{R_2 + R_4}{R_2 R_4} L_{12}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{L_1}{R_2} = \left(\frac{1}{R_4} + \frac{1}{R_2}\right) L_{12}$$

$$L_{12} = \frac{R_4}{R_2 + R_4} L_1$$