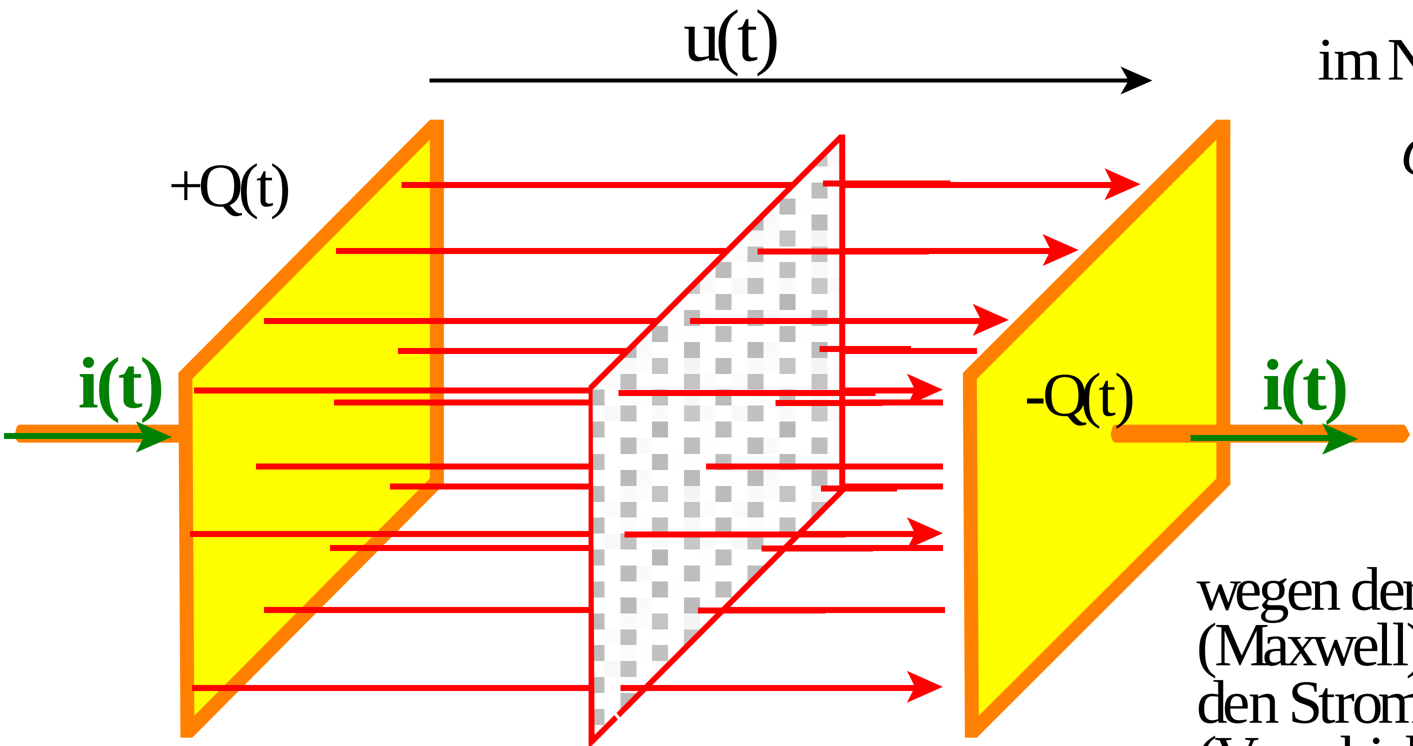
4.4 Zeitvorgänge in elektrisch nichtleitenden Medien

4.4.1 der Verschiebungsstrom



Leitungsstrom

$$i_L = \frac{dQ(t)}{dt}$$

im Nichtleiter gilt

$$Q(t) = \Psi_d(t)$$

wegen der Stromkontinuität (Maxwell) erhält man dann für den Strom im Nichtleiter (Verschiebungsstrom):

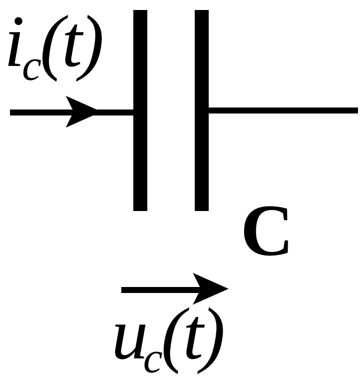
$$\boxed{i_V = i_L = \frac{d\Psi_d(t)}{dt} = \frac{dQ(t)}{dt}}$$

$$i_V = i_L = \frac{d\Psi_d(t)}{dt} = \frac{dQ(t)}{dt}$$

die Verschiebungsstrom-dichte wird dann

$$\boxed{\vec{J}_V = \frac{d\vec{D}(t)}{dt}}$$

für Kondensatoren ergeben sich als Konsequenzen:

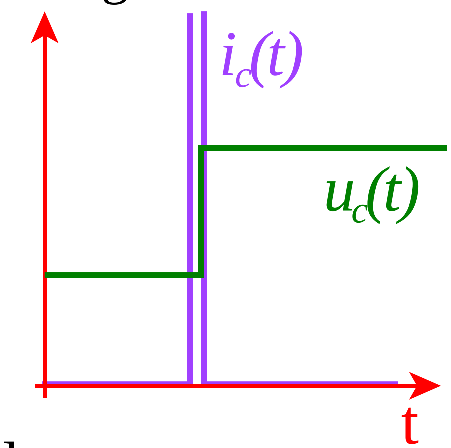


$$i_C(t) = \frac{dq}{dt}$$

$$q(t) = C\,u_C(t)$$

$$\boxed{i_C(t) = C\frac{du_C}{dt}}$$

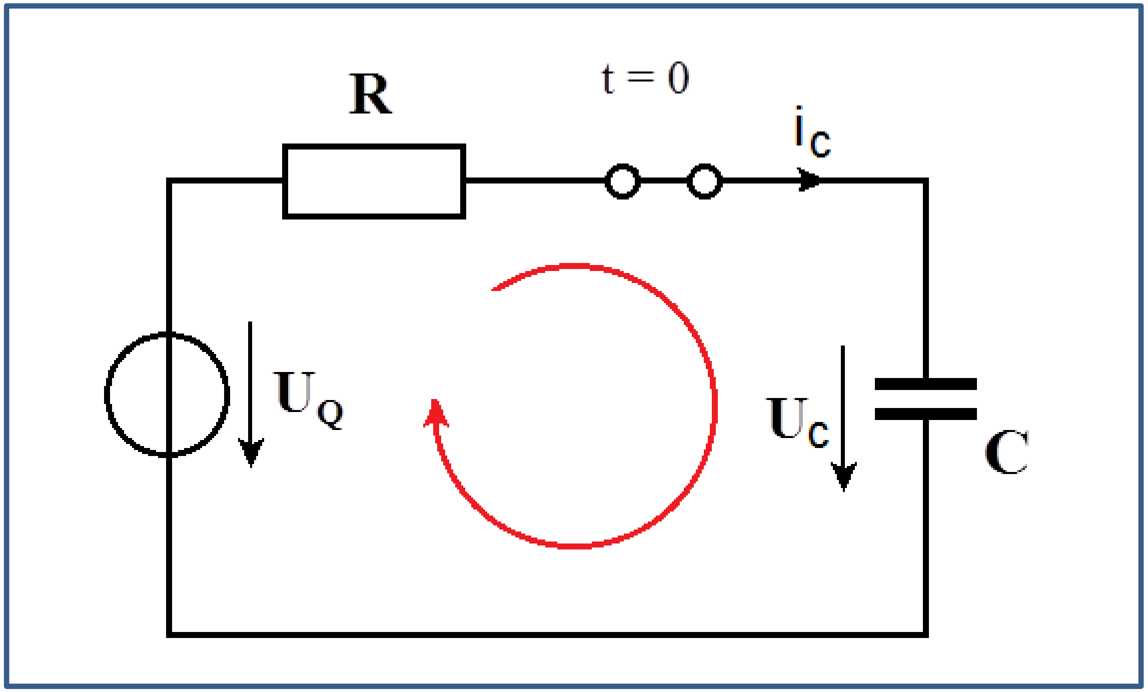
Energiezustandsänderungen



das Schaltgesetz:

$$\boxed{u_c(t-0) = u_c(t+0)}$$

4.4.2 Die Aufladung von Kondensatoren



$$U_Q = R \cdot i + u_c$$

$$i = C \frac{du_C}{dt}$$

$$U_Q = RC \frac{du_C}{dt} + u_C$$

lineare Differenzialgleichung 1. Ordnung mit konstanten Koeffizienten

Allgemein gilt:

$$X_\infty = \tau \cdot \frac{dx}{dt} + x \Rightarrow x(t) = X_\infty \cdot (1 - e^{-\frac{t}{\tau}}) + X_0 \cdot e^{-\frac{t}{\tau}}$$

oder

$$x(t) = X_\infty + (X_0 - X_\infty) \cdot e^{-\frac{t}{\tau}}$$

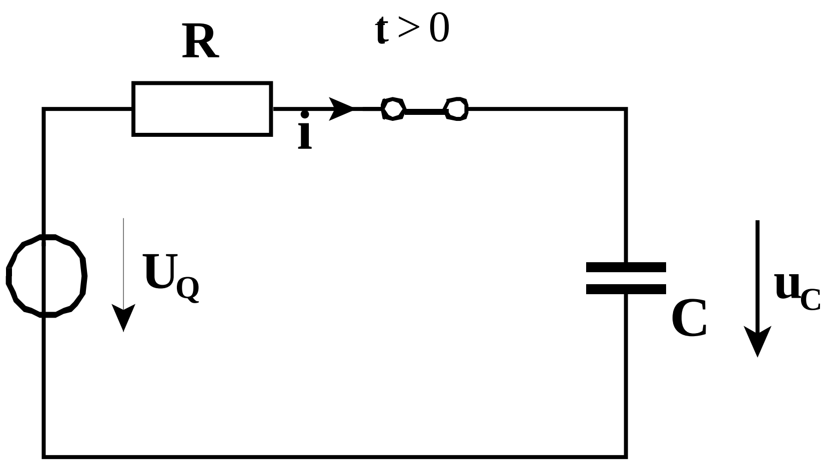
$$U_Q = RC \frac{du_C}{dt} + u_C$$

mit: $RC = \tau$ und $U_Q = U_{C\infty}$

$$U_{C\infty} = \tau \frac{du_C}{dt} + u_C$$

$$\boxed{u_c(t) = U_{C\infty} + (U_{C0} - U_{C\infty}) \cdot e^{-\frac{t}{\tau}}}$$ oder $$\boxed{u_c(t) = U_{C\infty} \cdot (1 - e^{-\frac{t}{\tau}}) + U_{C0} \cdot e^{-\frac{t}{\tau}}}$$

Zur Info ausführliche Lösung (bis Folie 7):



Lösungsmethode:

- Trennung der Variablen

$$U_{C\infty}\, dt = \tau\, du_C + u_C\, dt$$

$$-\tau\, du_C = u_C\, dt - U_{C\infty}\, dt$$

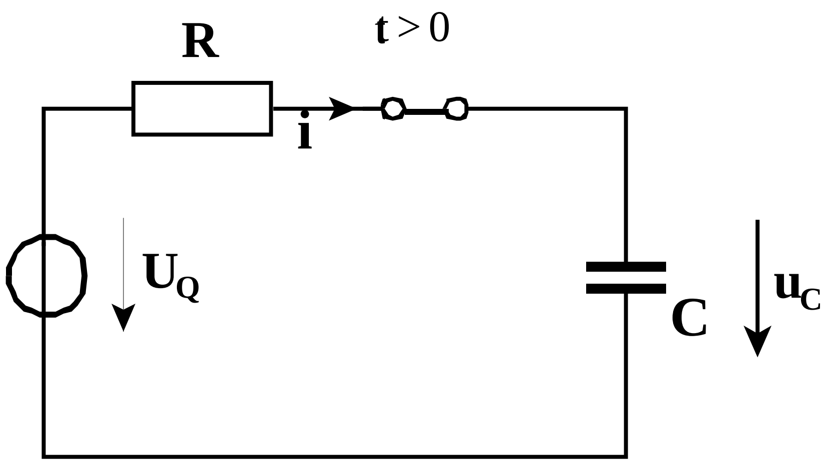
$$\frac{du_C}{u_C - U_{C\infty}} = -\frac{dt}{\tau}$$

$$\int \frac{du_C}{u_C - U_{C\infty}} = -\int \frac{dt}{\tau}$$

$$U_Q = RC\frac{du_C}{dt} + u_C$$

mit: $RC = \tau$ $\quad U_Q = U_{C\infty}$

$$U_{C\infty} = \tau\frac{du_C}{dt} + u_C$$



$$\int \frac{du_C}{u_C - U_{C\infty}} = -\int \frac{dt}{\tau}$$

$$\ln\left(u_C - U_{C\infty}\right) = -\frac{t}{\tau} + K'$$

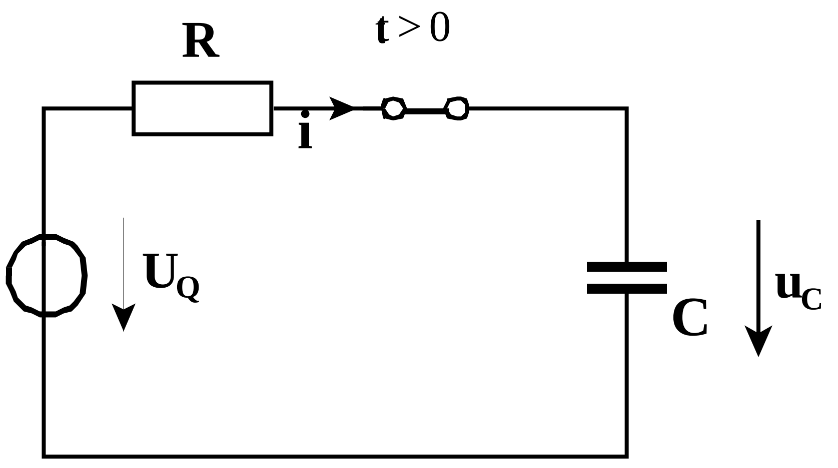
$$\ln\left(u_C - U_{C\infty}\right) = -\frac{t}{\tau} + \ln K$$

$$\ln \frac{\left(u_C - U_{C\infty}\right)}{K} = -\frac{t}{\tau}$$

$$\frac{u_C - U_{C\infty}}{K} = e^{-\frac{t}{\tau}}$$

und schließlich die allgemeine Lösung:

$$u_C = U_{C\infty} + K\,e^{-\frac{t}{\tau}}$$



$$u_C = U_{C\infty} + K\,e^{-\frac{t}{\tau}}$$

Anfangsbedingung in allgemeiner Form:

$$u_C(0-0) = u_C(0+0) = u_{C0} = U_{C\infty} + K\,e^{-\frac{0}{\tau}}$$

und damit wird die Integrationskonstante

$$K = u_{C0} - U_{C\infty}$$

insgesamt ergibt sich

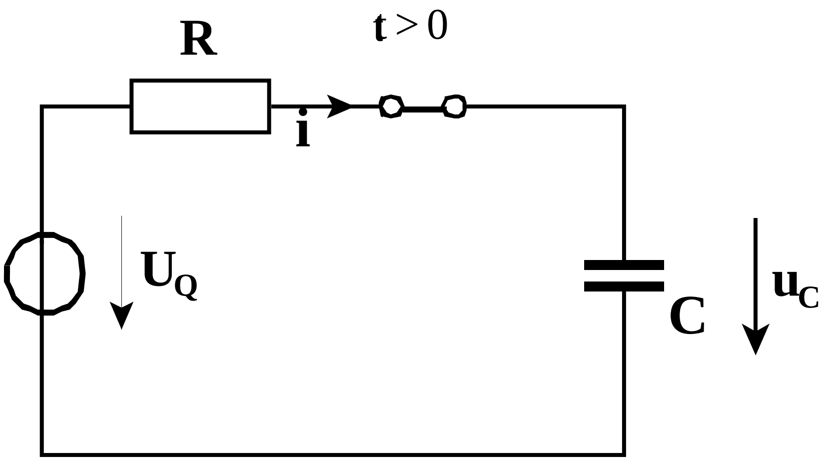
$$u_C = U_{C\infty} + (u_{C0} - U_{C\infty})\,e^{-\frac{t}{\tau}}$$

$$\boxed{u_c(t) = U_{C\infty} + (U_{C0} - U_{C\infty}) \cdot e^{-\frac{t}{\tau}}}$$

und für den Strom

τ=R*C

$$i = C\frac{du_C}{dt} = \left(u_{C0} - U_{C\infty}\right) C\, e^{-\frac{t}{\tau}} (-\frac{1}{\tau}) = \frac{U_{C\infty} - u_{C0}}{R} e^{-\frac{t}{\tau}}$$



$$u_C = U_{C\infty} + (u_{C0} - U_{C\infty})\, e^{-\frac{t}{\tau}}$$

$$i = \frac{U_{C\infty} - u_{C0}}{R}\, e^{-\frac{t}{\tau}}$$

Spezialfall:

Mit $U_{c\infty} = U_Q$ und $u_c(0) = 0$ ergibt sich:

$$u_c = U_Q(1 - e^{-\frac{t}{\tau}})$$

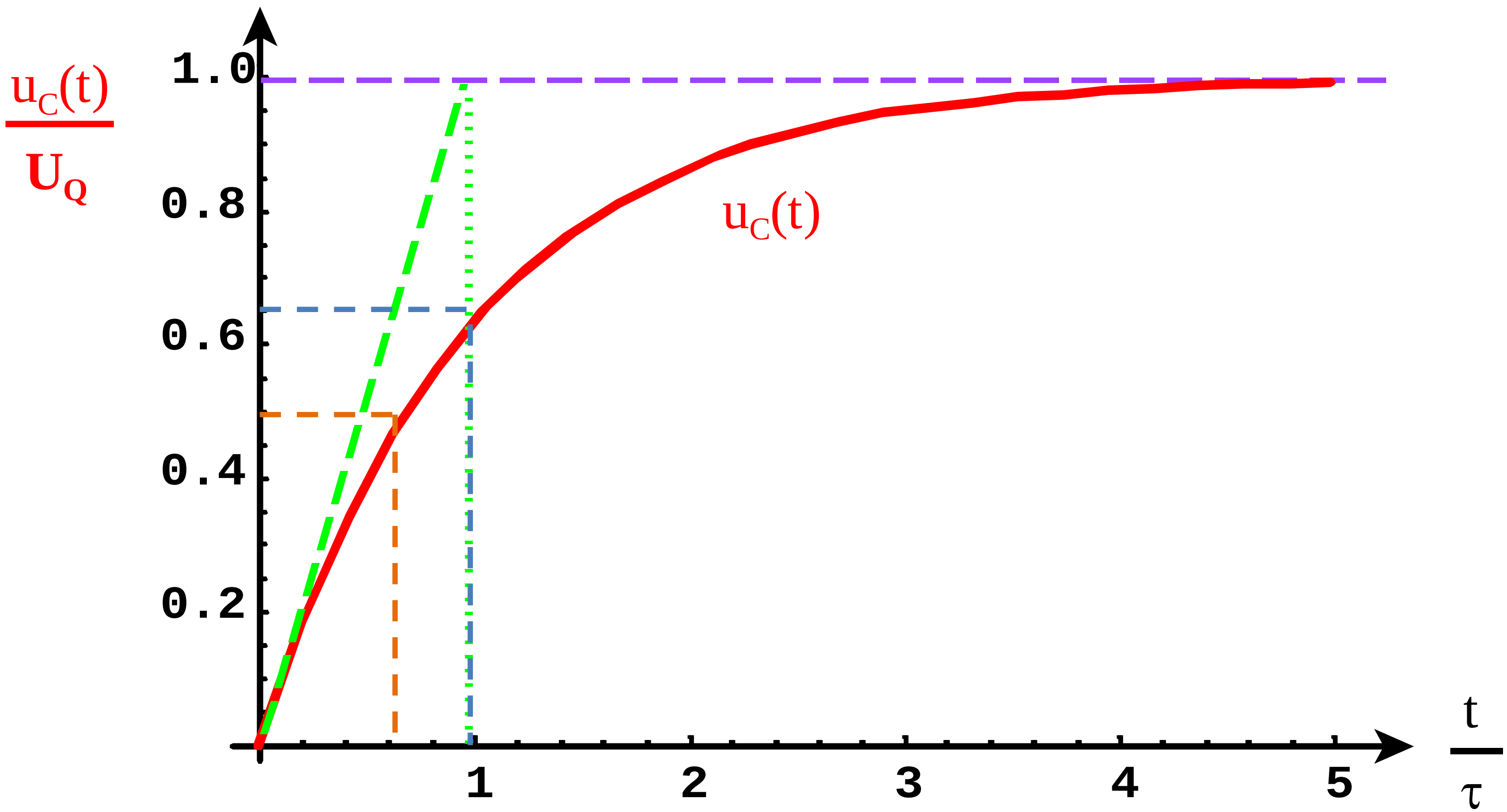
und für den Strom

$$i = C\frac{du_C}{dt} = -U_Q C\, e^{-\frac{t}{\tau}}(-\frac{1}{\tau}) = \frac{U_Q}{R} e^{-\frac{t}{\tau}}$$

$$\boxed{u_c = U_Q(1-e^{\frac{-t}{\tau}})} \quad u_c(3\tau) = U_Q(1-e^{\frac{-3\tau}{\tau}}) = 0{,}95U_Q \qquad u_c(5\tau) = 0{,}993U_Q$$

$$u_c = \frac{U_Q}{2} = U_Q(1-e^{\frac{-t}{\tau}}) \Rightarrow \frac{1}{2} = 1-e^{\frac{-t}{\tau}} \Rightarrow t = -\tau \cdot \ln(\frac{1}{2}) = 0{,}693\tau$$

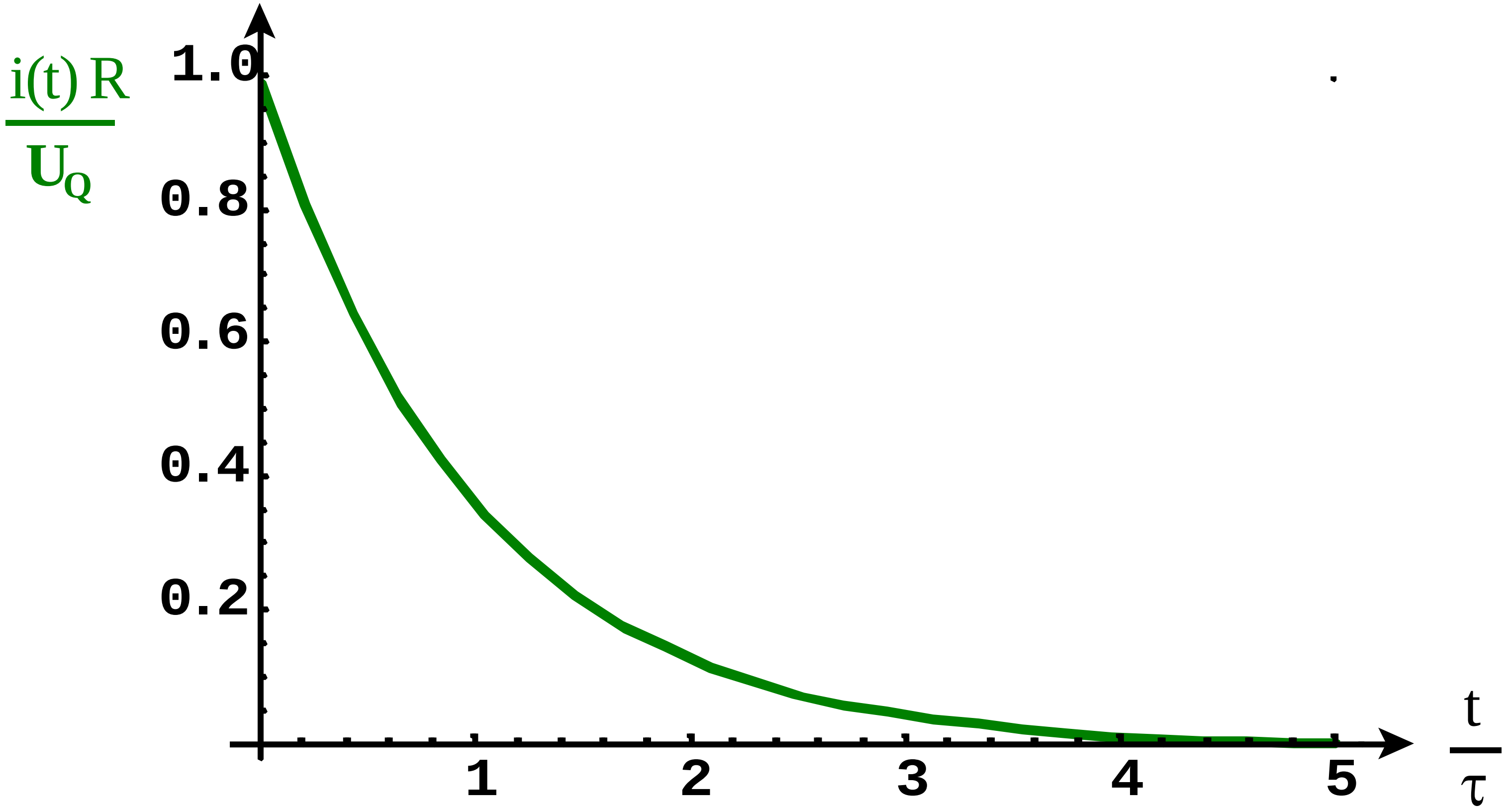
$$u_c(t=\tau) = U_Q(1-e^{\frac{-\tau}{\tau}}) = 0{,}632 \cdot U_Q$$



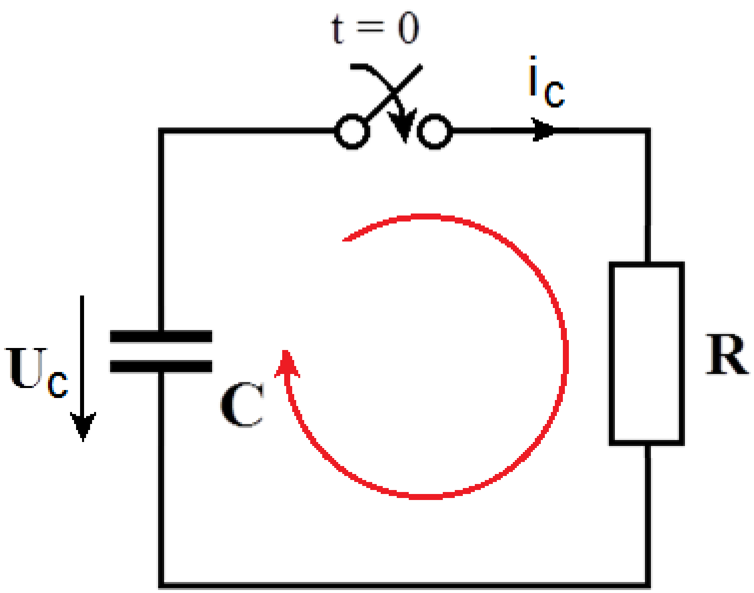
Kurvendiskussion:

$$u_C = U_Q(1 - e^{-\frac{t}{\tau}})$$

$$i = \frac{U_Q}{R} e^{-\frac{t}{\tau}}$$



4.4.3 die Entladung von Kondensatoren



$$R \cdot i_c - u_c = 0$$

$$i = -C \frac{du_C}{dt}$$

$$RC \frac{du_C}{dt} + u_C = 0$$

lineare homogene Differenzialgleichung 1. Ordnung mit konstanten Koeffizienten

Allgemein gilt:

$$X_\infty = \tau \cdot \frac{dx}{dt} + x \quad \Rightarrow \quad x(t) = X_\infty \cdot (1 - e^{-\frac{t}{\tau}}) + X_0 \cdot e^{-\frac{t}{\tau}}$$

oder

$$x(t) = X_\infty + (X_0 - X_\infty) \cdot e^{-\frac{t}{\tau}}$$

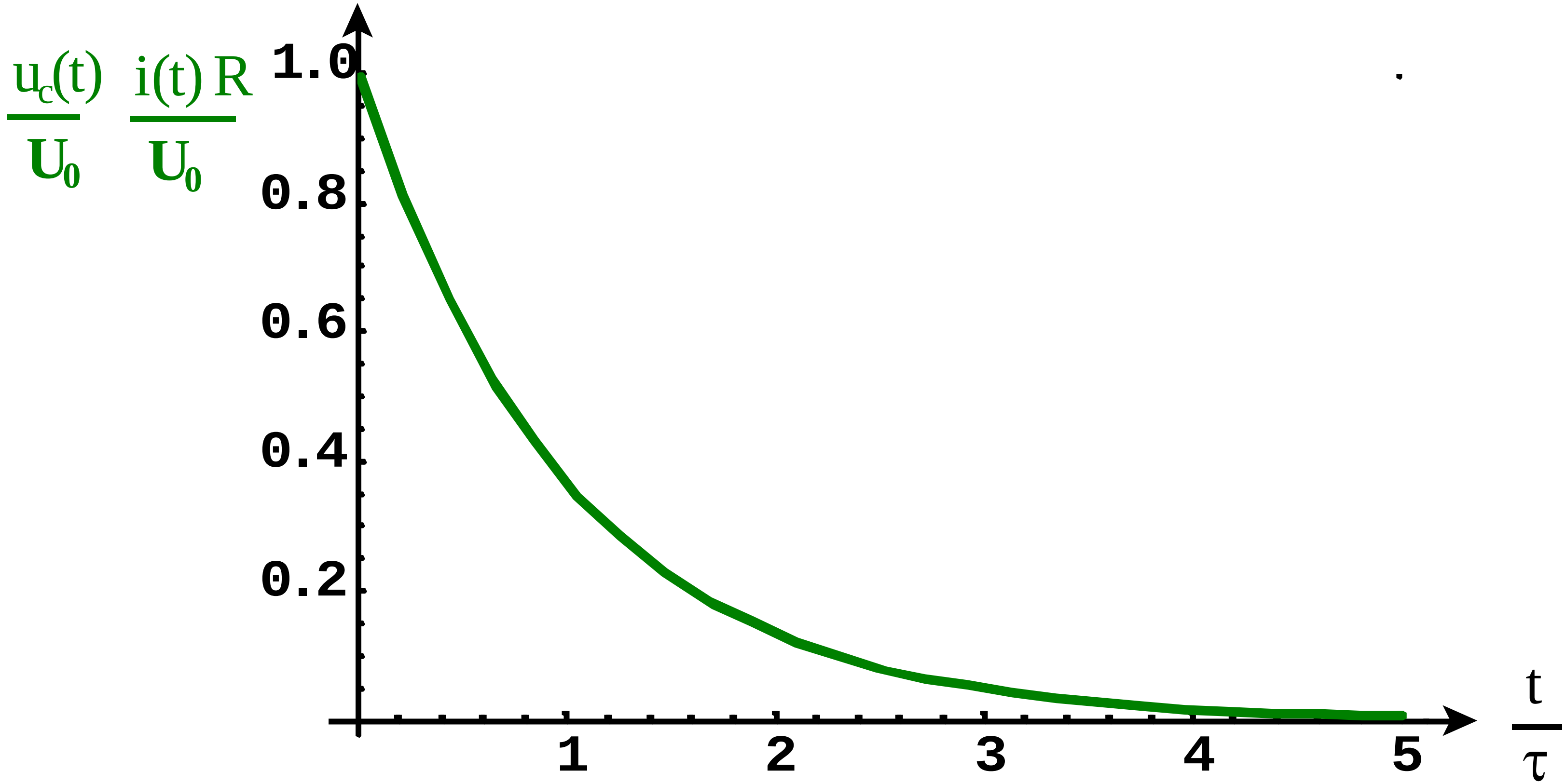
$$RC \frac{du_C}{dt} + u_C = 0$$

Abkürzung: $\boldsymbol{RC = \tau \quad 0 = U_{c\infty}}$

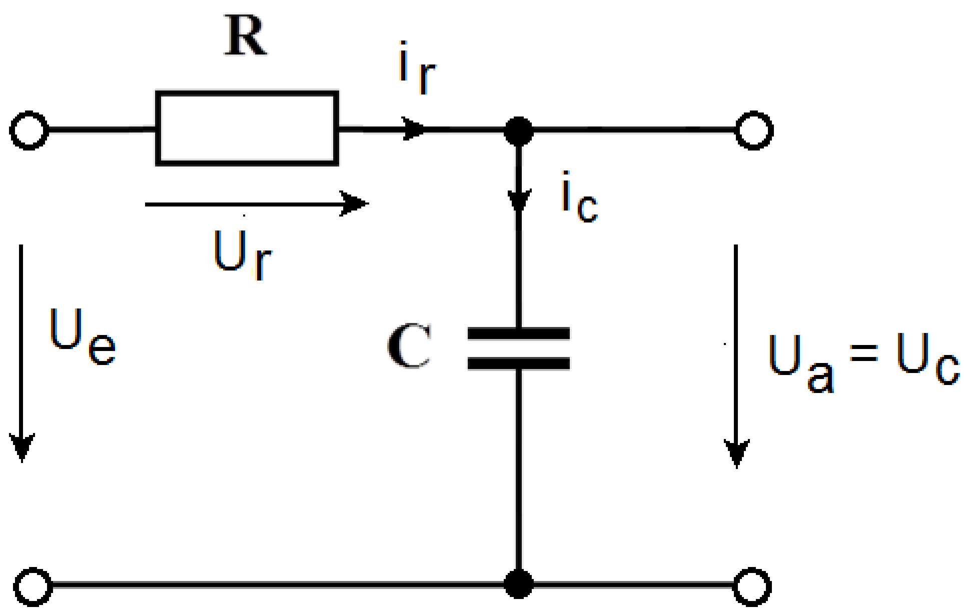
$$\tau \frac{du_C}{dt} + u_C = 0$$

$$u_C = U_0 \, e^{-\frac{t}{\tau}} \qquad i = \frac{U_0}{R} e^{-\frac{t}{\tau}}$$

Kurvendiskussion: $u_C = U_0\, e^{-\frac{t}{\tau}}$ $i = \frac{U_0}{R} e^{-\frac{t}{\tau}}$



Anwendung Integrierglied z.B. Laborversuch GET2 (Oszi)



$$i_c = C \cdot \frac{du_c}{dt}$$

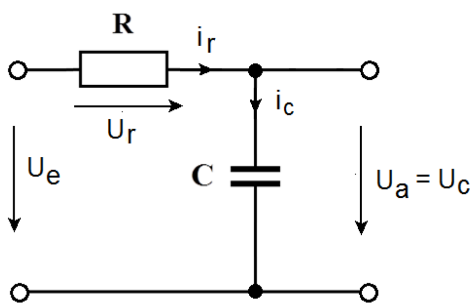
$$u_c = \frac{1}{C} \cdot \int i_c \cdot dt$$

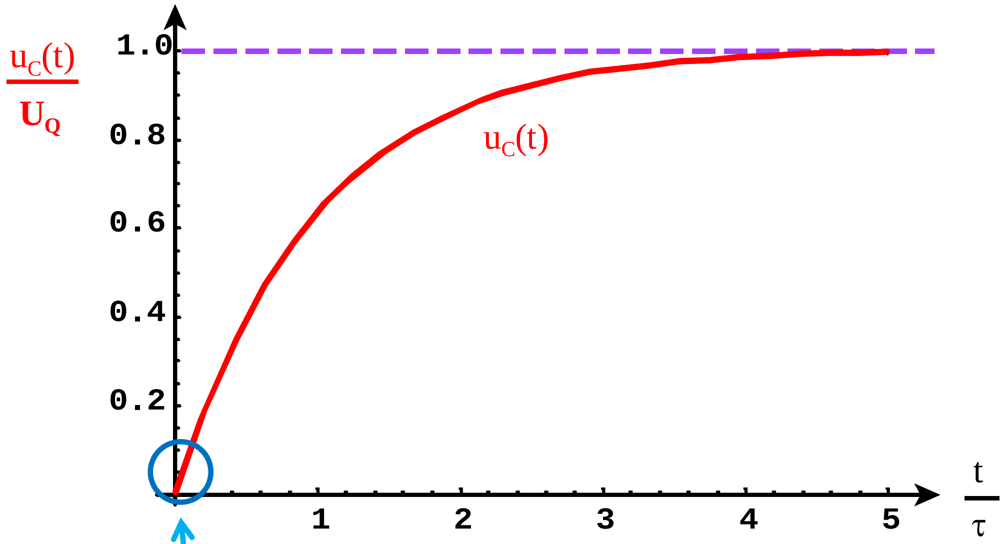
$$i_c = i_r \text{ und } i_r = \frac{u_r}{R}$$

$$u_c = \frac{1}{C} \cdot \int \frac{u_r}{R} \cdot dt = \frac{1}{R \cdot C} \cdot \int u_r \cdot dt$$

wenn $u_c \ll u_r$ dann gilt $u_e \approx u_r$

$$\boxed{u_a = \frac{1}{R \cdot C} \cdot \int u_e \cdot dt}$$





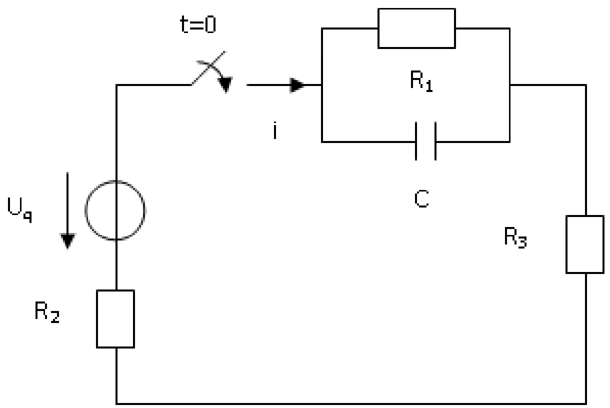
$$u_c = \frac{1}{C} \cdot \int \frac{u_r}{R} \cdot dt = \frac{1}{R \cdot C} \cdot \int u_r \cdot dt$$

wenn $u_c \ll u_r$ dann gilt $u_e \approx u_r$

$$\boxed{u_a = \frac{1}{R \cdot C} \cdot \int u_e \cdot dt}$$

gilt, wenn $t \ll \tau$

Beispiel:



$U_q = 12\ \mathrm{V}$

$R_1 = 2\ \mathrm{k\Omega}$

$R_2 = 100\ \Omega$

$R_3 = 100\ \Omega$

$C = 100\ \mathrm{pF}$

	t=0	t→∞
i	$\dfrac{U_q}{R_2 + R_3}$	$\dfrac{U_q}{R_2 + R_3 + R_1}$
u_c	0	$U_q \dfrac{R_1}{R_1 + R_2 + R_3}$

$$\boxed{u_C = U_{C\infty} + (u_{C0} - U_{C\infty})\, e^{-\frac{t}{\tau}}}$$

$$u_c = U_q \frac{R_1}{R_1 + R_2 + R_3}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$i = \frac{U_q}{R_1 + R_2 + R_3}\left(1 - e^{-\frac{t}{\tau}}\right) + \frac{U_q}{R_2 + R_3}\left(e^{-\frac{t}{\tau}}\right)$$