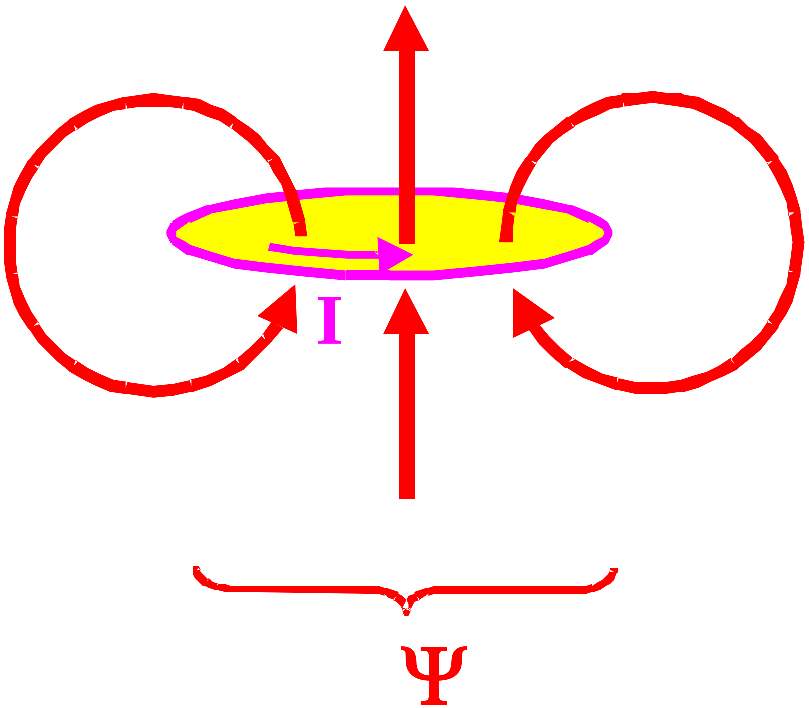
6.2 Selbstinduktion und Induktivität

- Grundbegriffe



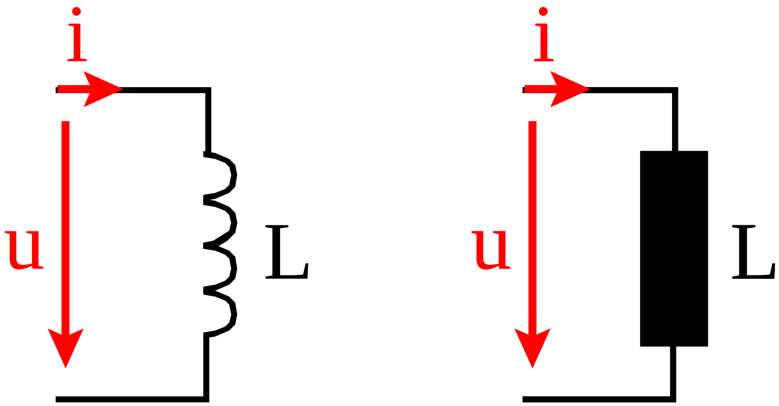
$$I \to H \to B \to \Phi \to \Psi$$

$$\Psi = L\,I$$

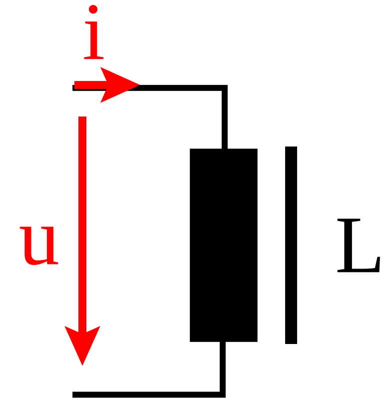
Induktivität: $\boxed{L = \dfrac{\Psi}{I}}$

$$\boxed{[L] = \frac{[\Psi]}{[I]} = 1\frac{\mathrm{Vs}}{\mathrm{A}} = 1\mathrm{H}}$$

- Schaltzeichen für Induktivitäten



lineare Spule

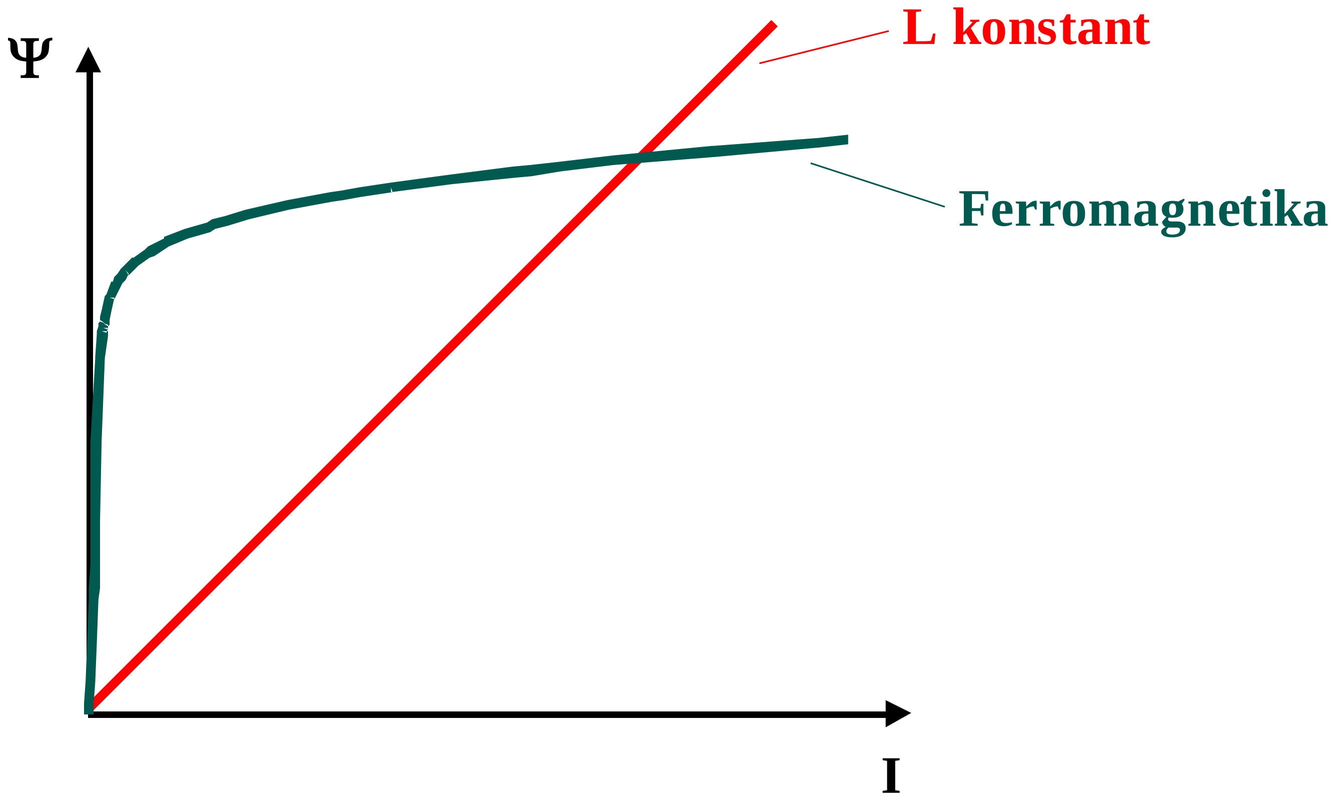


nichtlineare Spule

- Strom - Spannungsgleichung

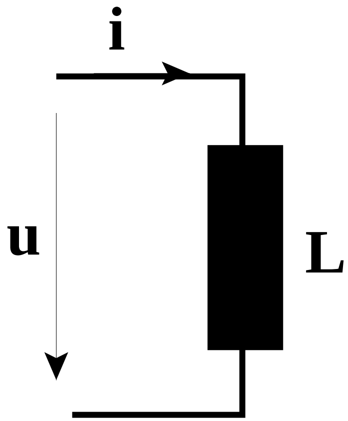
$$u = +\frac{d\Psi}{dt}$$

für L konstant : $$u = +L\frac{d\,i}{dt}$$



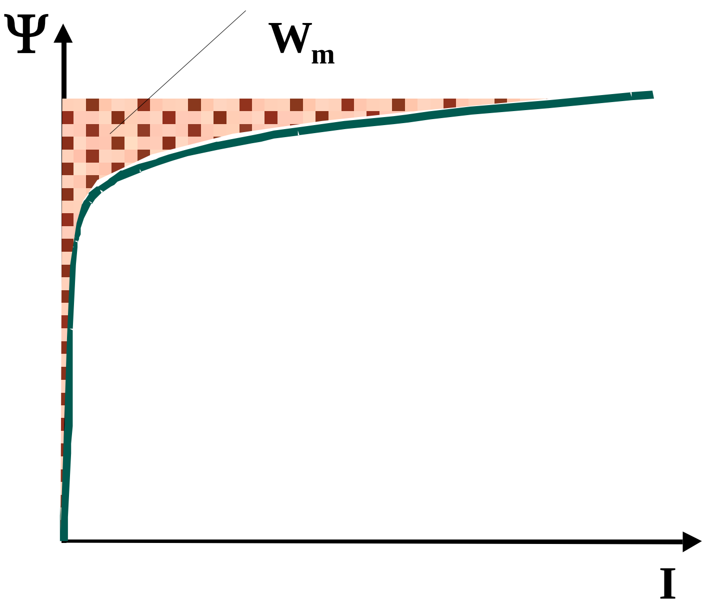
differentielle oder dynamische Induktivität: $$\boxed{L_d = \frac{d\Psi}{dI}}$$

- die in Induktivitäten gespeicherte magnetische Energie



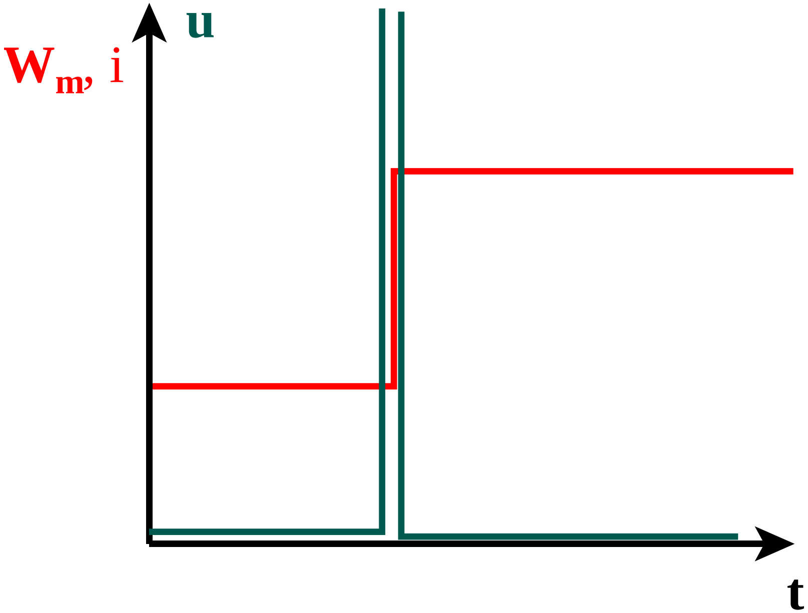
$$W_m = \int_0^\infty p_{el}\, dt = \int_0^\infty u\, i\, dt$$

für L konstant : $u = +L \dfrac{d\,i}{dt}$



$$\boxed{W_m = \int_0^\infty i\, L \frac{di}{dt} dt = \int_0^I i\, L\, di = \frac{L}{2} I^2}$$

- das Schaltgesetz an Induktivitäten



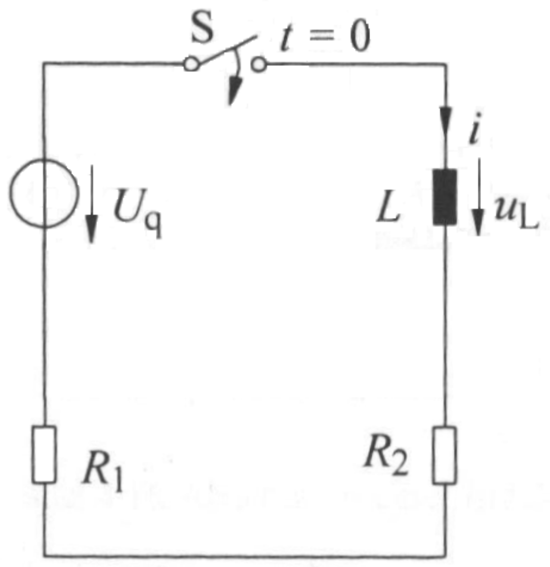
$$W_m = \frac{L}{2} I^2$$

für L konstant :

$$u = + L \frac{d\,i}{dt}$$

$$\boxed{i_L(t-0) = i_L(t+0)}$$

- Anschalten einer Induktivität an Gleichspannung – Berechnung des Stromes:



$$0 = \frac{d\Psi}{dt} + R_2 i + R_1 i - U_q \qquad \Psi = L\,I$$

$$\frac{U_q}{R_1 + R_2} = \frac{L}{R_1 + R_2} * \frac{di}{dt} + i$$

$$I_\infty = \tau * \frac{di}{dt} + i$$

Lineare DGL 1. Ordnung und 1. Grades, Lösung:

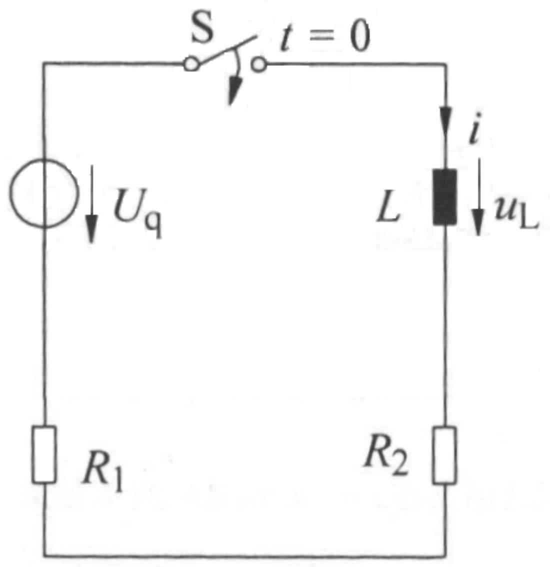
$$\tau = \frac{L}{R_{ers}} = \frac{L}{R_1 + R_2}$$

$$x(t) = X_\infty (1 - e^{-\frac{t}{\tau}}) + X_0 e^{-\frac{t}{\tau}}$$

Mit der Anfangsbedingung I(t=0) = 0 wird:

$$\boxed{i(t) = \frac{U_q}{R_1 + R_2}(1 - e^{-\frac{t}{\tau}})}$$

- Anschalten einer Induktivität an Gleichspannung – Berechnung der Spannung:



Anfangsbedingung: i(0) = 0

$$0 = -U_q + u_L + i(0)(R_1 + R_2)$$

$$u_L(0) = U_q$$

Endbedingung:

$$u_L = L\frac{di}{dt} = 0$$

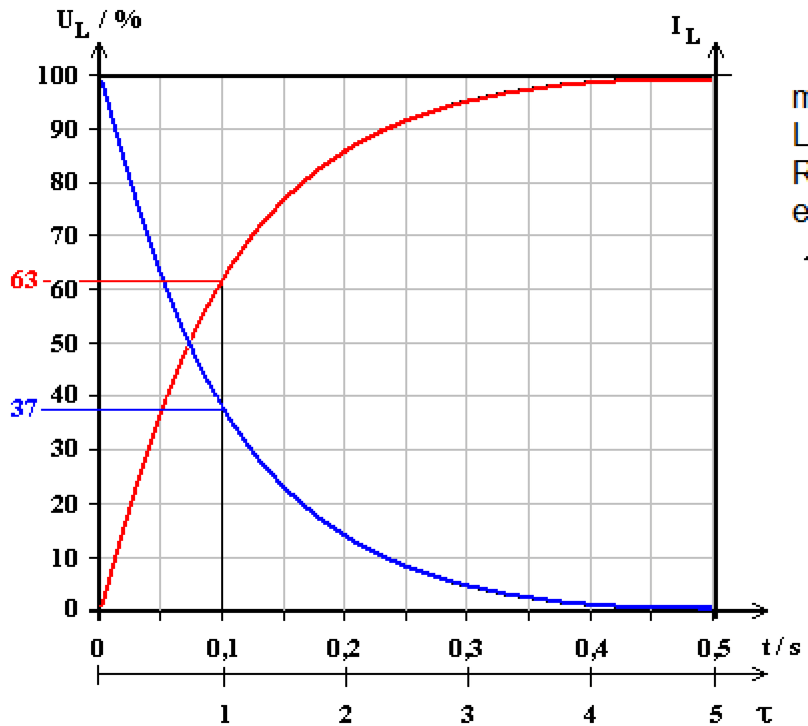
$$x(t) = X_\infty(1 - e^{-\frac{t}{\tau}}) + X_0 e^{-\frac{t}{\tau}}$$

$$\boxed{u_L(t) = U_q e^{-\frac{t}{\tau}}}$$

- Anschalten einer Induktivität an Gleichspannung – Zeitverlauf von Strom und Spannung:

$$\boxed{i(t) = \frac{U_q}{R_1 + R_2}(1 - e^{-\frac{t}{\tau}})}$$

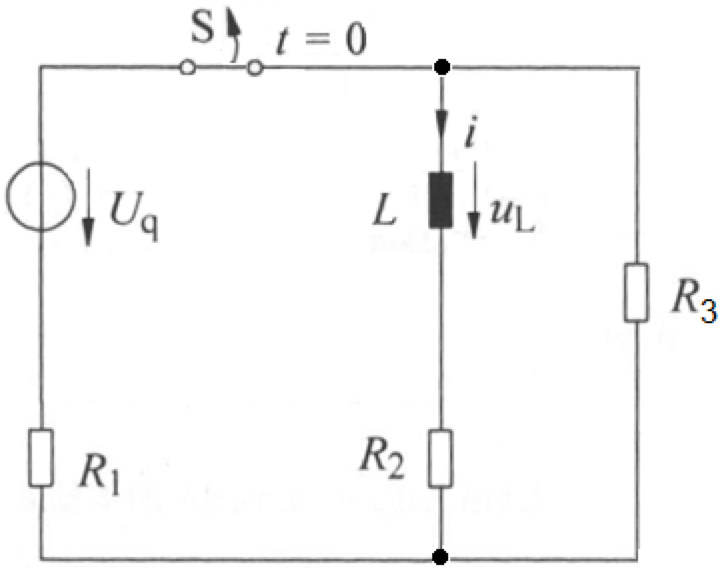
$$\boxed{u_L(t) = U_q e^{-\frac{t}{\tau}}}$$



mit
L=1H und
R1=R2=5 Ω
ergibt sich:

$$\tau = \frac{L}{R} = 0{,}1\ s$$

- Abschalten einer Induktivität bei Gleichspannung – Berechnung des Stromes:



Werte: Uq=24V R1=R2=1Ω   R3=1kΩ
L=1H

Anfangsbedingung: I (t=-0) = I(t=+0)

$$I(t=-0)=\frac{U_q}{R_1+R_2\,||\,R_3}*\frac{R_3}{R_2+R_3}$$

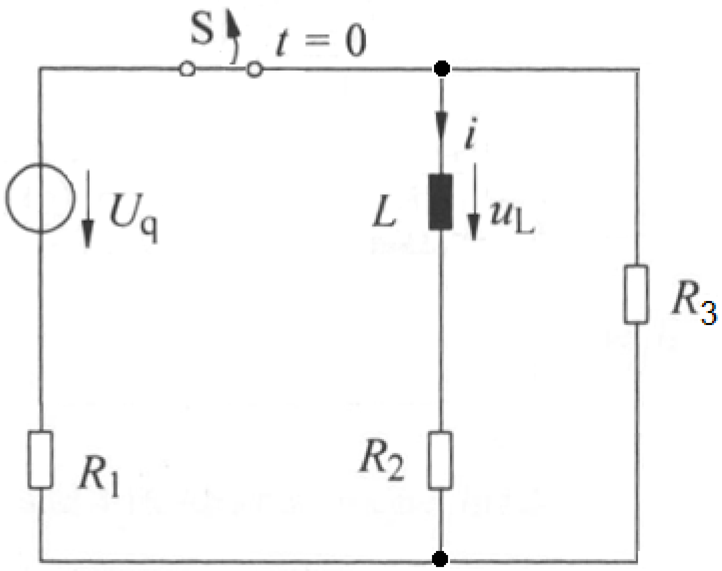
$$I(t=0)=11{,}994\,A$$

Endbedingung: $I_\infty = 0$

$$x(t)=X_\infty(1-e^{-\frac{t}{\tau}})+X_0e^{-\frac{t}{\tau}}$$

$$\boxed{i(t)=\frac{U_q}{R_1+R_2\,||\,R_3}*\frac{R_3}{R_2+R_3}*e^{-\frac{t}{\tau}}}$$

- Abschalten einer Induktivität bei Gleichspannung – Berechnung der Spannung:



Werte: Uq=24V R1=R2=1Ω R3=1kΩ L=1H

Anfangsbedingung: I (t=-0) = I(t=+0)

$$0 = U_L(t=0) + I(t=0) * (R_2 + R_3)$$

$$U_L(t=0) = -I(t=0) * (R_2 + R_3)$$

$$I(t=0) = \frac{U_q}{R_1 + R_2 \| R_3} * \frac{R_3}{R_2 + R_3}$$

$$U_L(t=0) = -\frac{U_q}{R_1 + R_2 \| R_3} * R_3$$

$$U_L(t=0) = -12006V$$

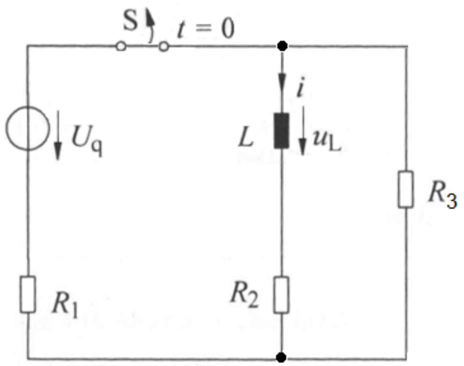
Endbedingung:

$$U_L(t \to \infty) = 0$$

$$x(t) = X_\infty (1 - e^{-\frac{t}{\tau}}) + X_0 e^{-\frac{t}{\tau}}$$

$$\boxed{u_L(t) = -\frac{U_q}{R_1 + R_2 \| R_3} * R_3 * e^{-\frac{t}{\tau}}}$$

- Abschalten einer Induktivität bei Gleichspannung – Zeitverlauf von Strom und Spannung:



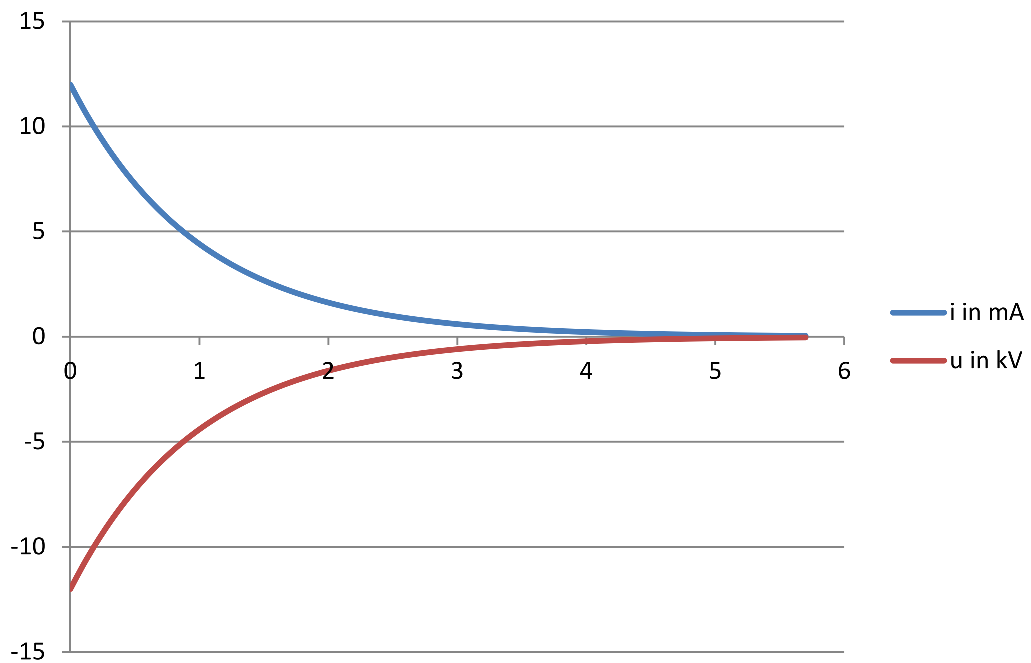
$$i(t) = \frac{U_q}{R_1 + R_2 \parallel R_3} * \frac{R_3}{R_2 + R_3} * e^{-\frac{t}{\tau}}$$

$$u_L(t) = -\frac{U_q}{R_1 + R_2 \parallel R_3} * R_3 * e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_{ers}} = \frac{L}{R_2 + R_3} = 0{,}999ms$$

$$I(t=0) = 11{,}994A$$

$$U_L(t=0) = -12006V$$



Anwendung der Definitionsgleichung

$$L = \frac{\Psi}{I}$$

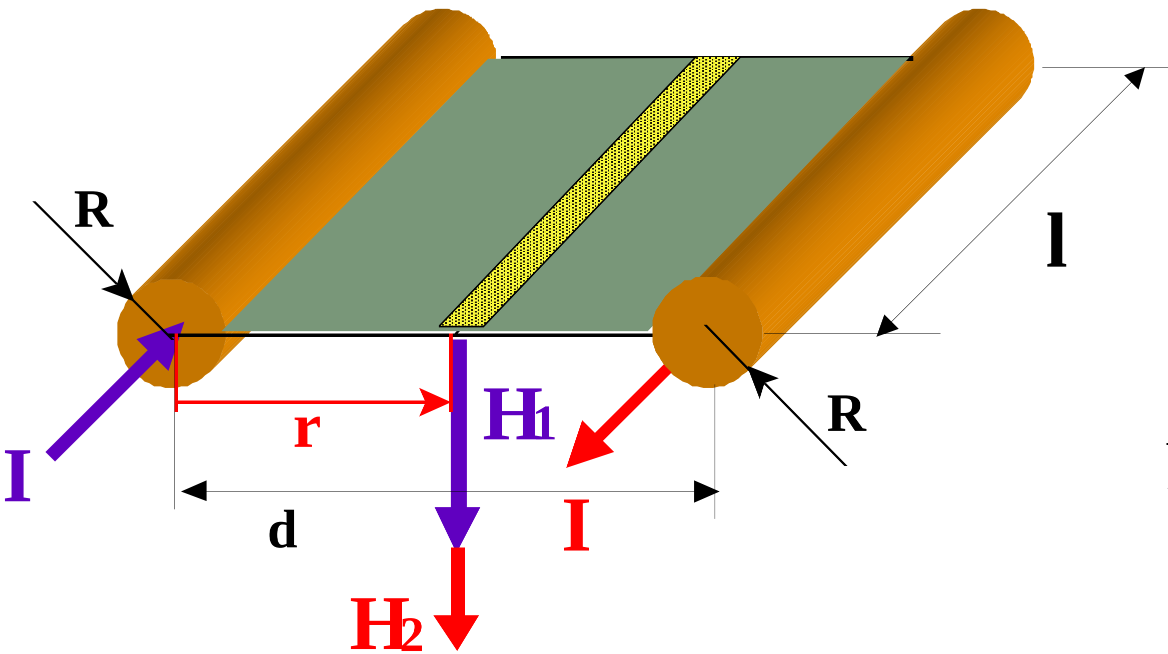
Lösungsweg:

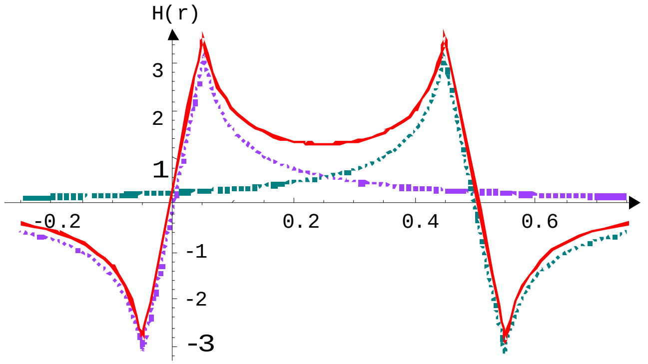
1. Annahme eines Stromes durch die Leiterschleife oder Spule, deren Induktivität zu berechnen ist.
2. Ermitteln des mit der Leiterschleife bzw. Spule verketteten Flusses.
3. Division des so berechneten verketteten Flusses durch den verursachenden Strom.

- **Beispiel:** äußere Induktivität einer Paralleldrahtleitung

- Berechnung des Magnetflusses zwischen den Leitern

$$H = H_1 + H_2 = \frac{I}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right)$$



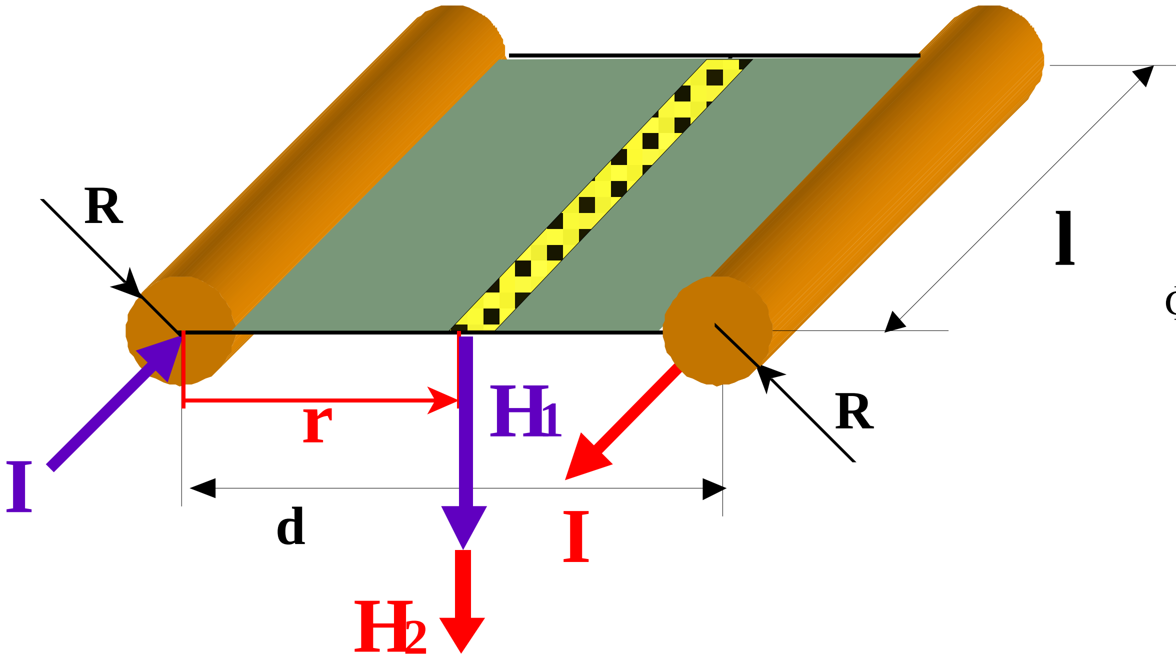


$$\vec{B} = \mu\,\vec{H} = -\frac{I\,\mu}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right)\vec{e}_z$$

$$\Phi = \int_A \vec{B}\,d\vec{A}$$

$$H_1 = \frac{I}{2\pi r} \qquad H_2 = \frac{I}{2\pi(d-r)}$$

$$\Phi = \int_A \frac{-I\,\mu}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right)\vec{e}_z\,d\vec{A}$$



$$\Phi = \int_A \frac{-I\,\mu}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right)\vec{e}_z\, d\vec{A}$$

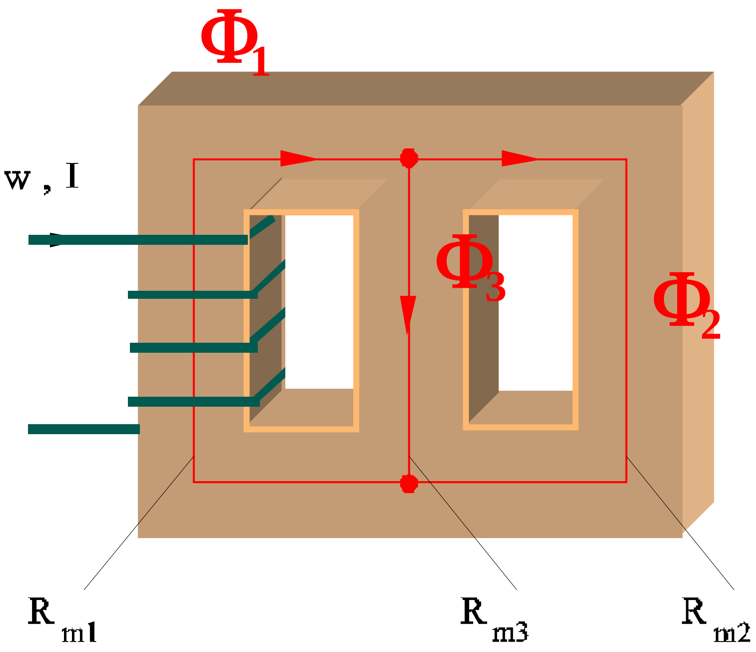
$$d\vec{A} = -dr\, l\, \vec{e}_z$$

$$\Phi = \int_R^{d-R} \frac{I\,\mu}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right) l\, dr \quad = \frac{I\,\mu\, l}{\pi} \ln\frac{d-R}{R}$$

$$L = \frac{\Psi}{I} = \frac{\Phi}{I} = \frac{\mu\, l}{\pi} \ln\frac{d-R}{R}$$

die Induktivitätsbemessungsgleichung

$$L = \frac{w\Phi_1}{I} = \frac{w}{I}\frac{\Theta}{R_{mers}} = \frac{w}{I}\frac{wI}{R_{mers}}$$



$$\boxed{L = \frac{w^2}{R_{mers}} = w^2 \Lambda_{ers}}$$

- Beispiel: Trafokern

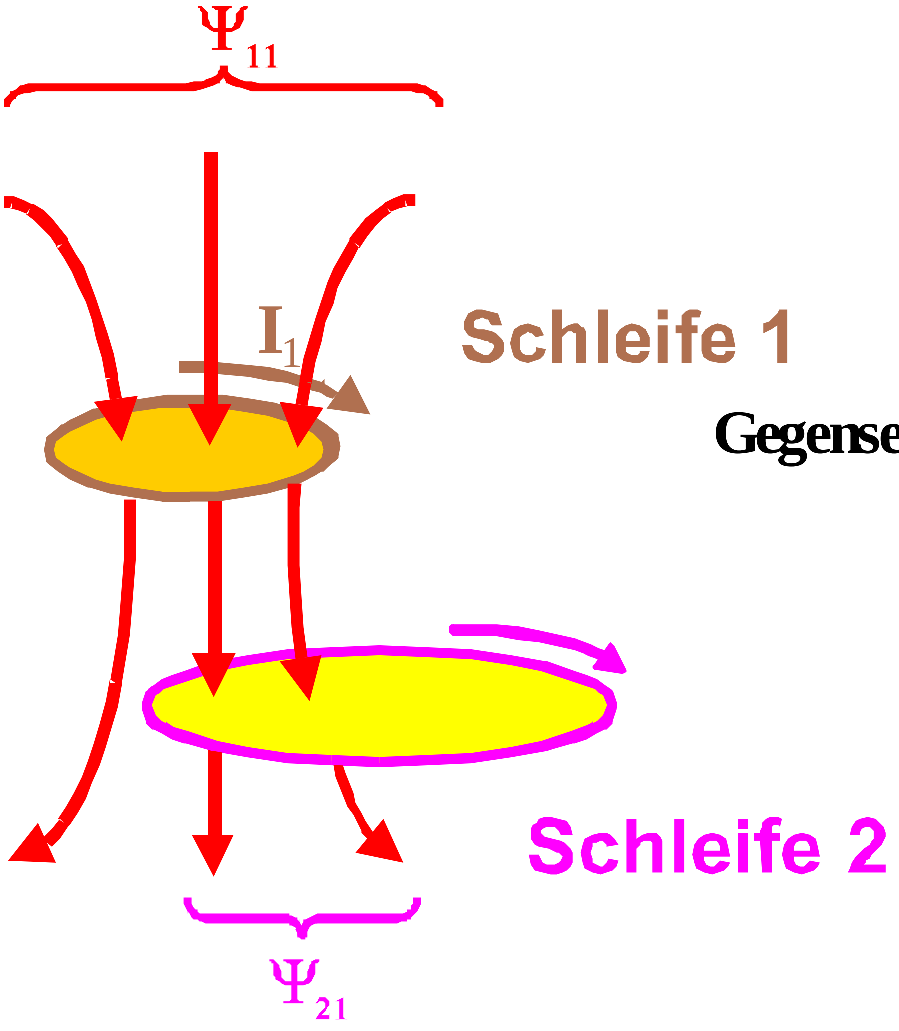
$$R_{ers} = R_{m1} + \frac{1}{\frac{1}{R_{m2}} + \frac{1}{R_{m3}}} = R_{m1} + \frac{R_{m2}\,R_{m3}}{R_{m2} + R_{m3}}$$

$$L = \frac{\Psi}{I} \qquad L = \frac{w\Phi_1}{I}$$

$$L = \frac{w^2}{R_{mers}} = \frac{w^2}{R_{m1} + \dfrac{R_{m2}R_{m3}}{R_{m2} + R_{m3}}}$$

6.3 Gegeninduktion und gegenseitige Induktivität (Gegeninduktivität)

6.3.1 Grundbegriffe



$$\boxed{I_1 \to H \to B \to \Phi_{11} \to \Phi_{21}}$$

$$\Psi_{21} = L_{21} I_1 = M I_1$$

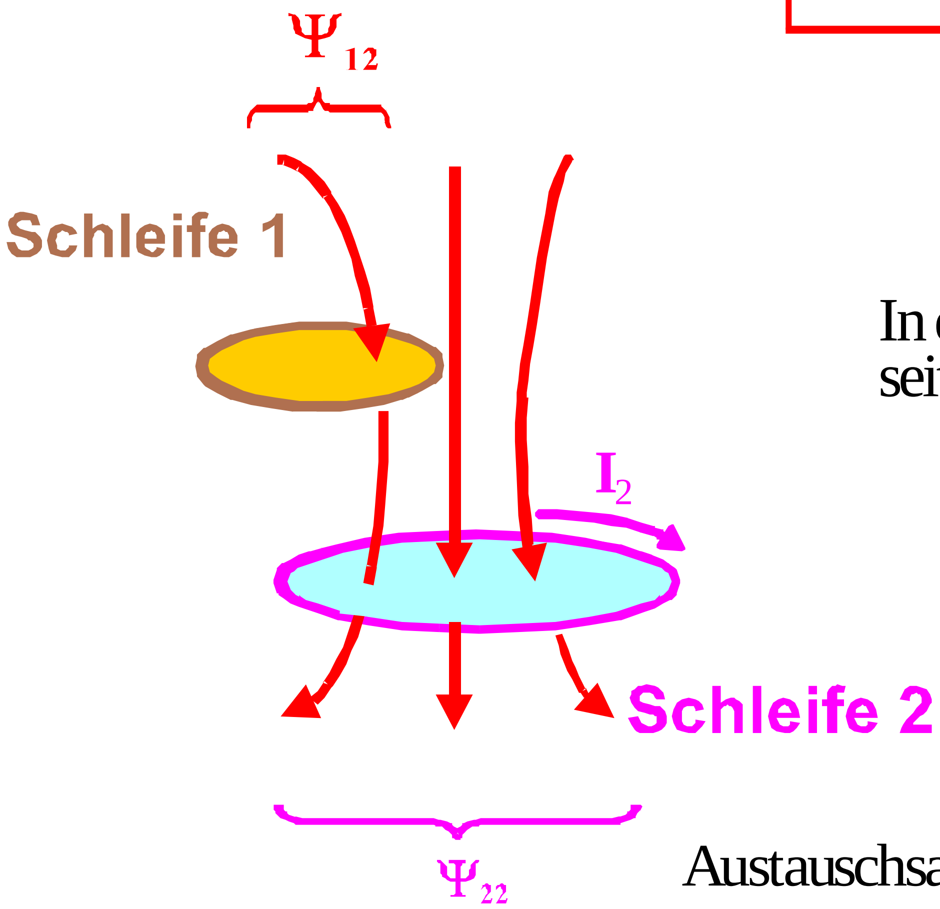
Gegenseitige Induktivität (Gegeninduktivität)

$$\boxed{L_{21} = M = \frac{\Psi_{21}}{I_1}}$$

$$[M] = \frac{[\Psi]}{[I]} = 1\frac{\mathrm{Vs}}{\mathrm{A}} = 1\mathrm{H}$$

Umgekehrt ergibt sich:

$$\boxed{I_2 \rightarrow H \rightarrow B \rightarrow \Phi_{22} \rightarrow \Phi_{12} \rightarrow \Psi_{12}}$$



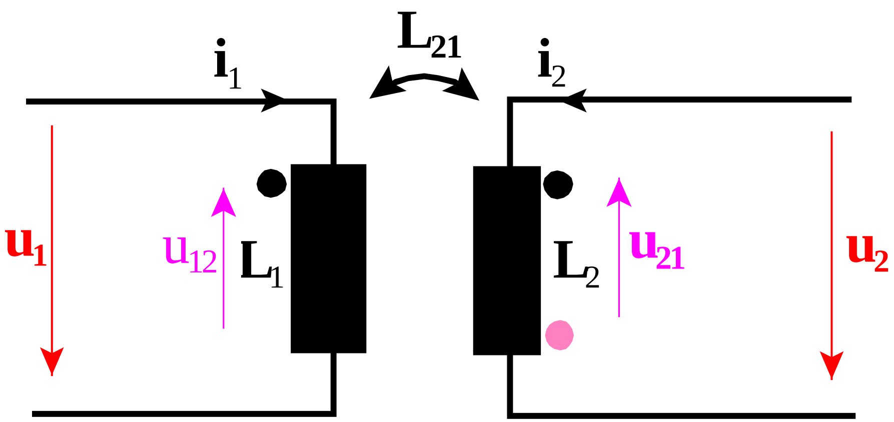
$$\Psi_{12} = L_{12}\, I_2 = M\; I_2$$

In diesem Fall erhält man für die gegenseitige Induktivität (Gegeninduktivität)

$$\boxed{L_{12} = M = \frac{\Psi_{12}}{I_2}}$$

Austauschsatz: $\boxed{L_{12} = L_{21} = M = \frac{\Psi_{21}}{I_1} = \frac{\Psi_{12}}{I_2}}$

- *Schaltzeichen für Gegeninduktivitäten*



- *Strom - Spannungsgleichung*

$$u = + \frac{d\Psi}{dt}$$

für L_{jk} konstant : $$\boxed{u_{jk} = L_{jk} \frac{d\,i_k}{dt}}$$

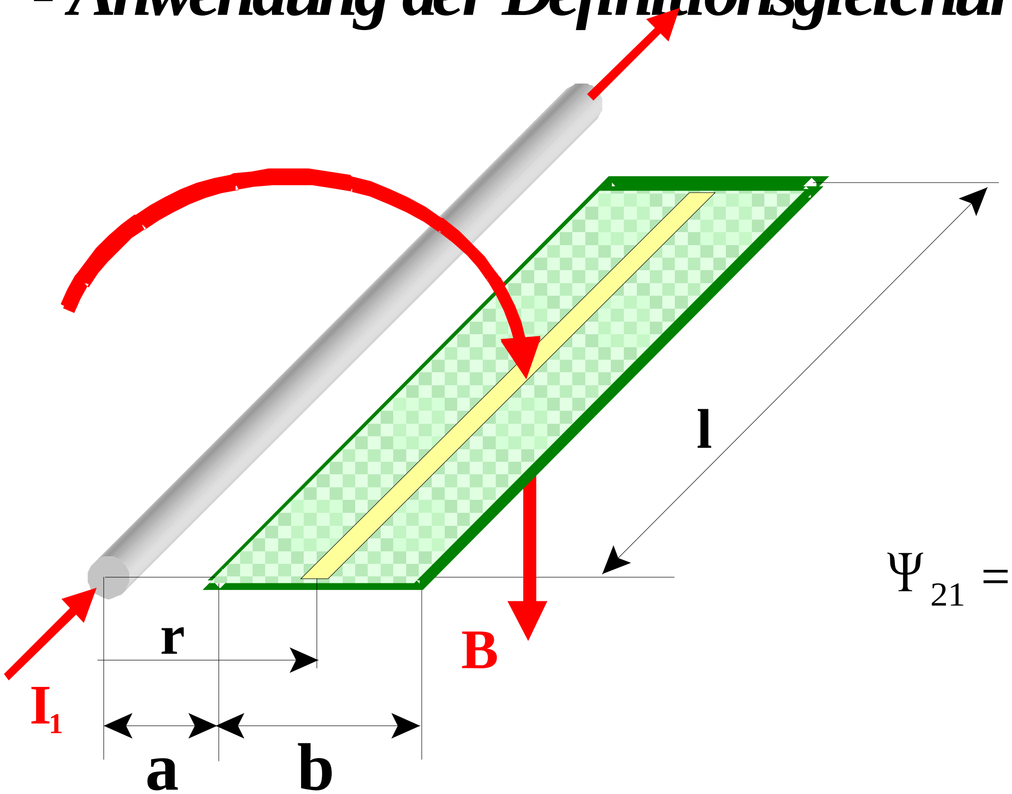
- *Gegeninduktivität in Stromkreisen*

● ● $$u_1 = +L_1 \frac{d\,i_1}{dt} + L_{12} \frac{d\,i_2}{dt} \qquad u_2 = +L_2 \frac{d\,i_2}{dt} + L_{21} \frac{d\,i_1}{dt}$$

● ● $$u_1 = +L_1 \frac{d\,i_1}{dt} - L_{12} \frac{d\,i_2}{dt} \qquad u_2 = +L_2 \frac{d\,i_2}{dt} - L_{21} \frac{d\,i_1}{dt}$$

6.3.2 Gegeninduktivitätsberechnung

- Anwendung der Definitionsgleichung



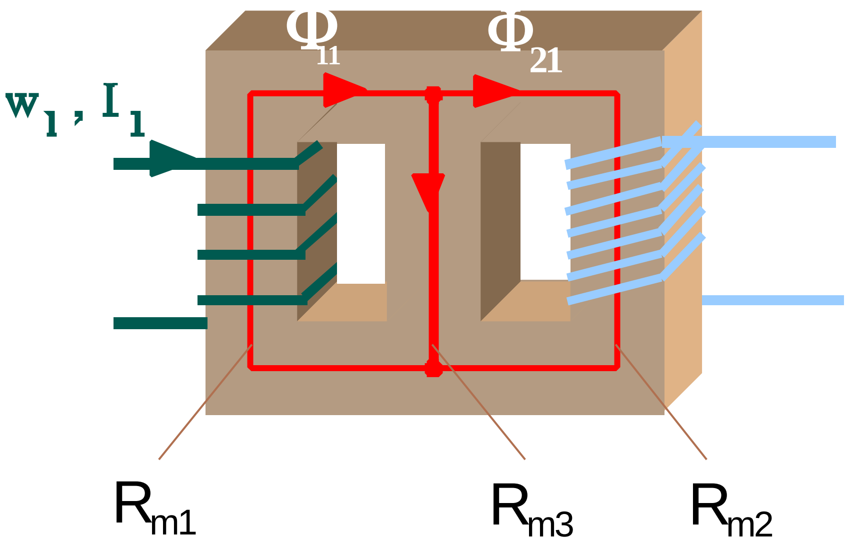
$$\boxed{L_{21} = \frac{\Psi_{21}}{I_1}}$$

$$\Psi_{21} = \Phi_{21} = \int_A \vec{B}\, d\vec{A} = \int_A \mu \vec{H}\, d\vec{A}$$

$$\Psi_{21} = \Phi_{21} = \int_a^{a+b} \frac{I_1 \mu}{2\pi} \frac{1}{r} l\, dr = \frac{I_1 \mu l}{2\pi} \ln \frac{a+b}{a}$$

$$L_{21} = \frac{\Psi_{21}}{I_1} = \frac{\Phi_{21}}{I_1} = \frac{\mu l}{2\pi} \ln \frac{a+b}{a}$$

- die Gegeninduktivitätsbemessungsgleichung



$$L_{21}=\frac{\Psi_{21}}{I_1} \qquad L_{21}=\frac{w_2\Phi_{21}}{I_1}$$

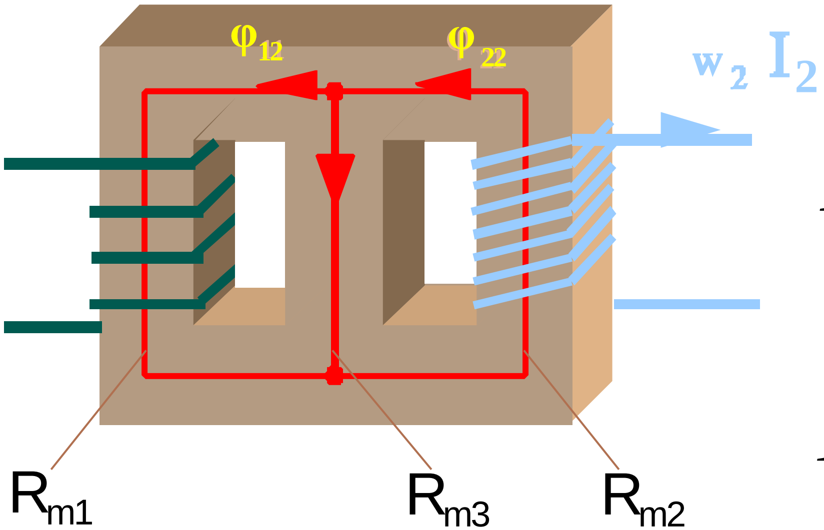
$$L_{21}=\frac{w_2 k_{21}\Phi_{11}}{I_1}=\frac{w_2 k_{21}}{I_1}\frac{\Theta_1}{R_{mers1}}$$

$$L_{21}=\frac{w_2 k_{21}}{I_1}\frac{w_1 I_1}{R_{mers1}}=k_{21}\frac{w_1 w_2}{R_{mers1}}$$

$$L_{21}=k_{21}\frac{w_1 w_2}{R_{mers1}}=k_{21}\frac{w_1 w_2}{R_{mers1}}\frac{w_1}{w_1}=k_{21}\frac{w_2}{w_1}L_1$$

$$L_1=\frac{w_1^2}{R_{mers1}}$$

$$L_{21} = k_{21}\frac{w_2}{w_1}L_1$$

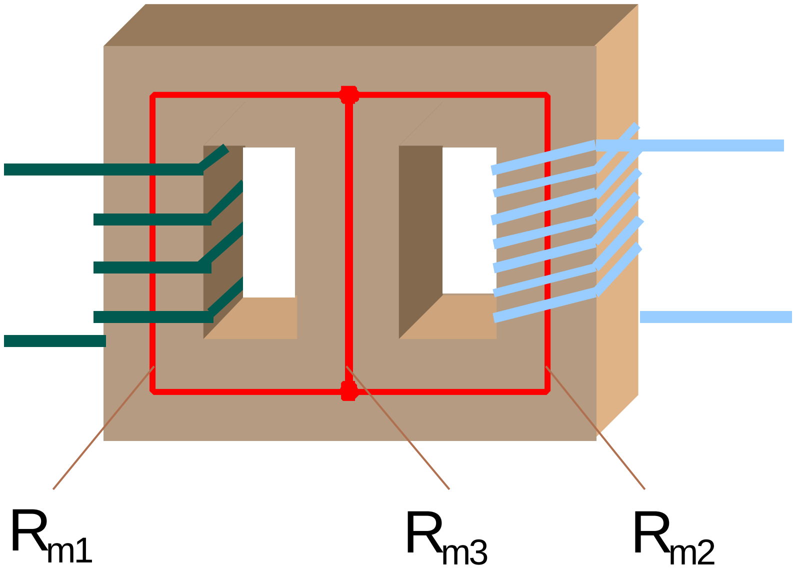


$$L_{12} = L_{21} = \frac{\Psi_{12}}{I_2} \qquad L_{21} = \frac{w_1\Phi_{12}}{I_2}$$

$$L_{21} = \frac{w_1 k_{12}\Phi_{22}}{I_2} = \frac{w_1 k_{12}}{I_2}\frac{\Theta_2}{R_{mers2}}$$

$$L_{21} = \frac{w_1 k_{12}}{I_2}\frac{w_2 I_2}{R_{mers2}} = k_{12}\frac{w_1 w_2}{R_{mers2}}$$

$$L_{21} = k_{12}\frac{w_1 w_2}{R_{mers2}} = k_{12}\frac{w_1 w_2}{R_{mers2}}\frac{w_2}{w_2} = k_{12}\frac{w_1}{w_2}L_2$$



$$L_{21} = k_{21}\frac{w_2}{w_1}L_1$$

$$L_{21} = k_{12}\frac{w_1}{w_2}L_2$$

$$L_{21}^2 = k_{12}k_{21}L_1L_2$$

$$L_{21} = \sqrt{k_{12}k_{21}L_1L_2} = \sqrt{k_{12}k_{21}}\sqrt{L_1L_2}$$

$$\boxed{k = \sqrt{k_{12}k_{21}}}$$

$$\boxed{L_{21} = k\sqrt{L_1L_2}}$$