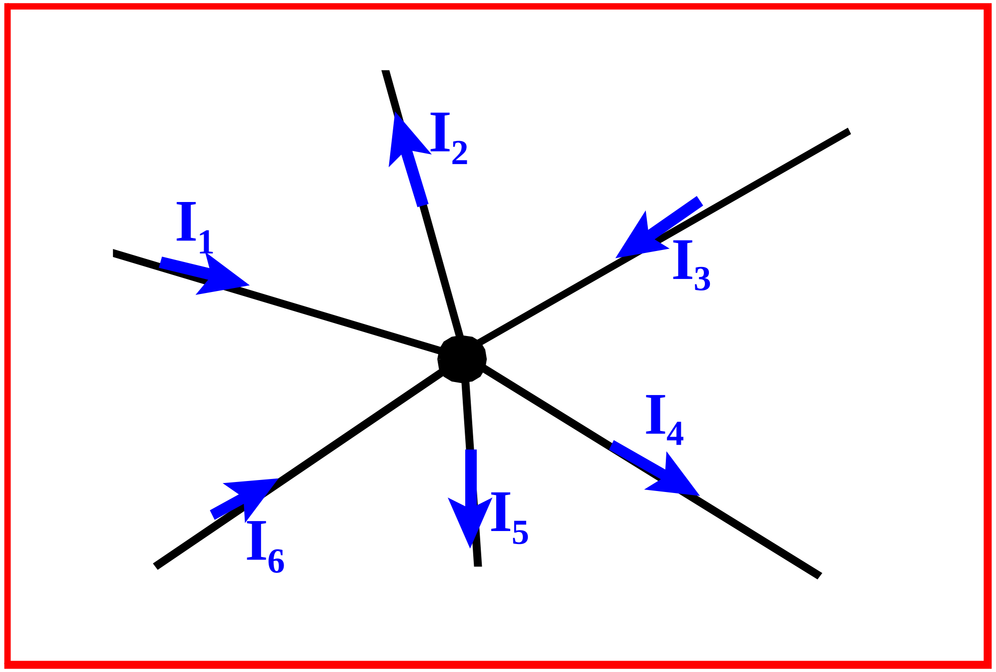
2. Vorgänge in elektrischen Netzwerken bei Gleichstrom

2.1 Der Knotensatz (1. Kirchhoff´scher Satz)



Ladungserhaltungssatz

$$\sum_i Q_i = konst.$$

$$\frac{d}{dt}\left(\sum_i Q_i\right) = 0$$

$$\sum_i \frac{dQ_i}{dt} = 0$$

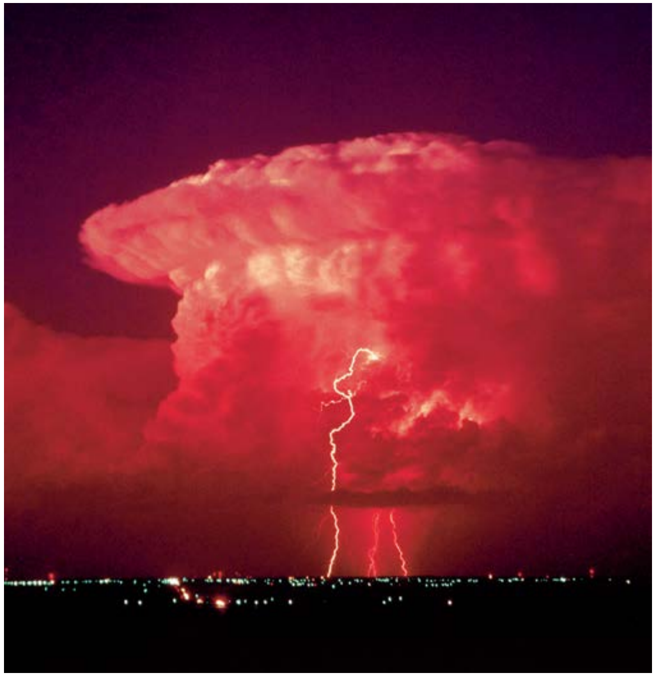
Knotensatz (1. Kirchhoff´scher Satz)

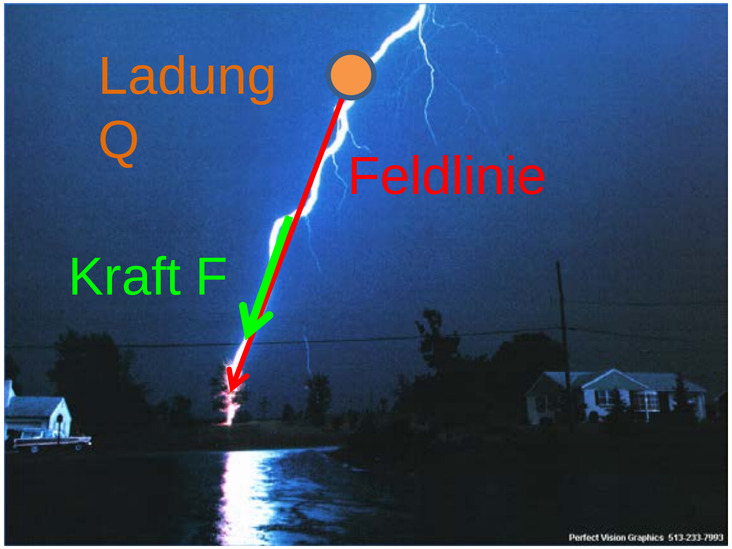
$$\sum_i I_{i\,vorzeichen} = 0$$

oder

$$\sum_i I_{i\uparrow} = \sum_i I_{i\downarrow}$$

2. 2 Kräfte auf Ladungen, Feldstärke und Spannung



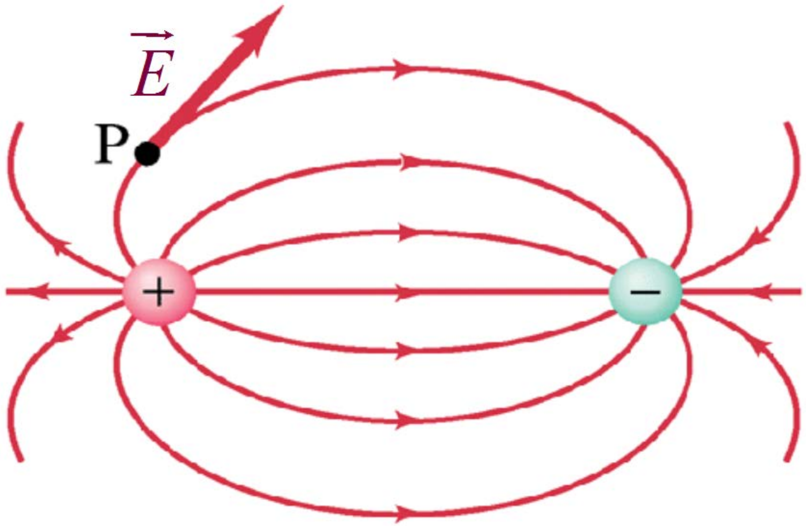


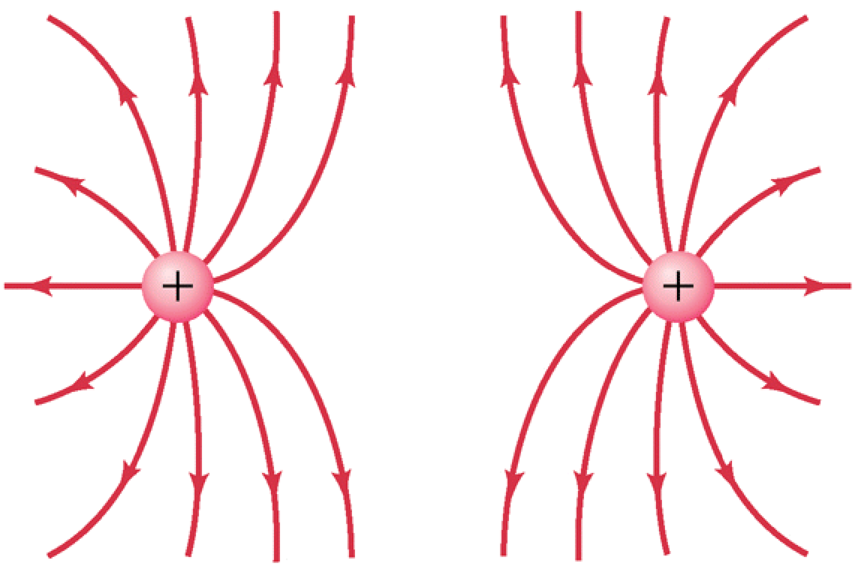
$$\vec{F} \sim Q$$

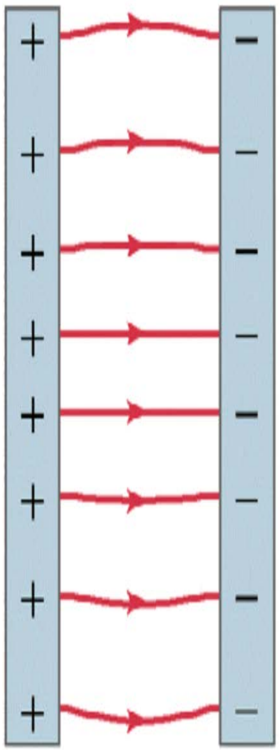
$$\boxed{\vec{F} = Q\,\vec{E}}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{Q}}$$

Beispiele für reale Feldbilder





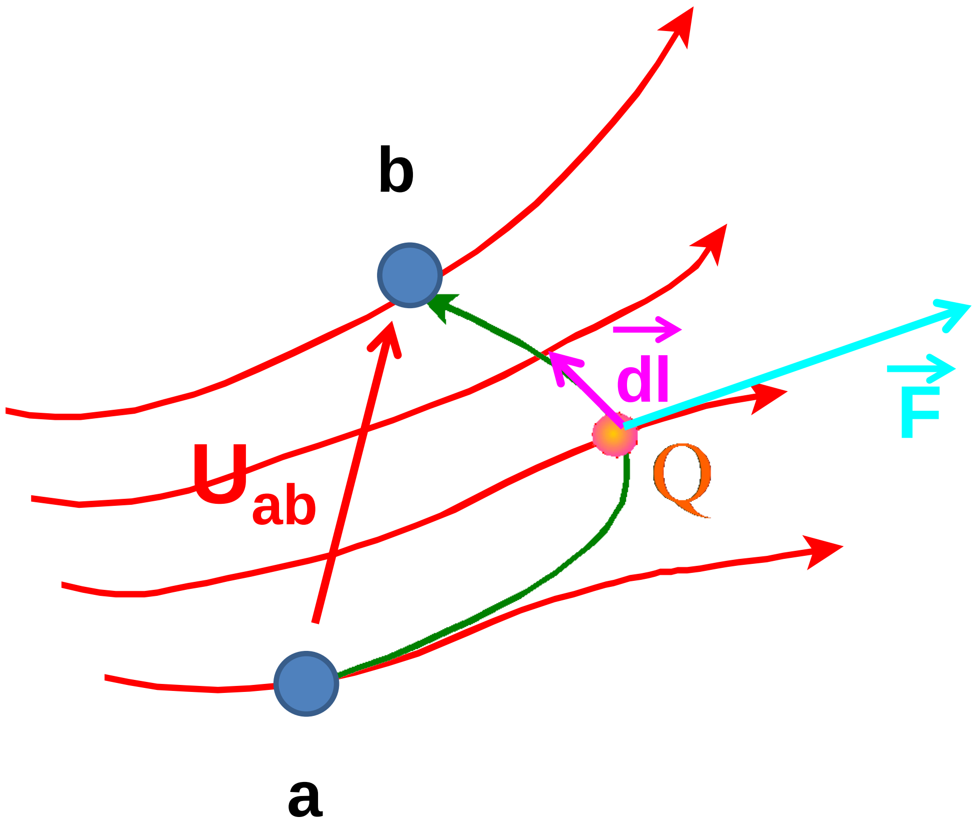


die elektrische Spannung:

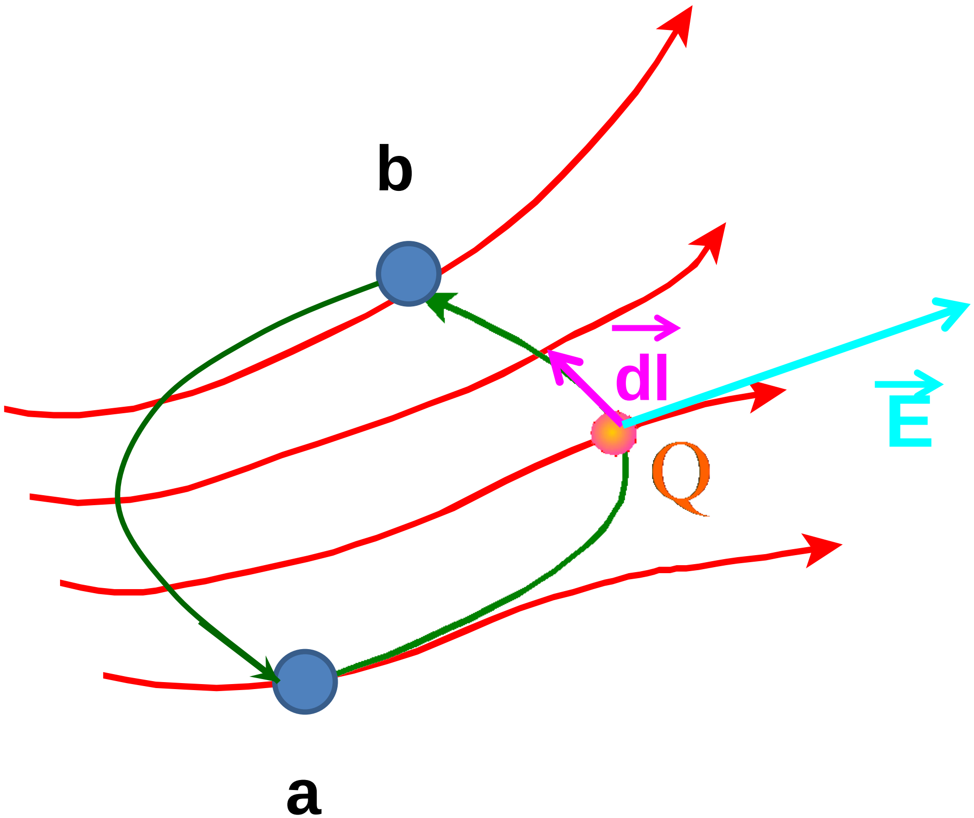
$$\Delta W_{el} = \int_a^b \vec{F}\, d\vec{l}$$

$$\vec{F} = Q\,\vec{E}$$

$$\Delta W_{el} = \int_a^b Q\,\vec{E}\, d\vec{l} = Q\int_a^b \vec{E}\, d\vec{l}$$



$$\frac{\Delta W_{el}}{Q} = \int_a^b \vec{E}\, d\vec{l} = U_{ab}$$



$$\Delta W_{el} = \int_a^b \vec{F}\, d\vec{l} = Q \int_a^b \vec{E}\, d\vec{l}$$

$$\boxed{\Delta W_{el\,umlauf} = 0}$$

Dann wird

$$\Delta W_{el\,umlauf} = Q \oint \vec{E}\, d\vec{l} = 0$$

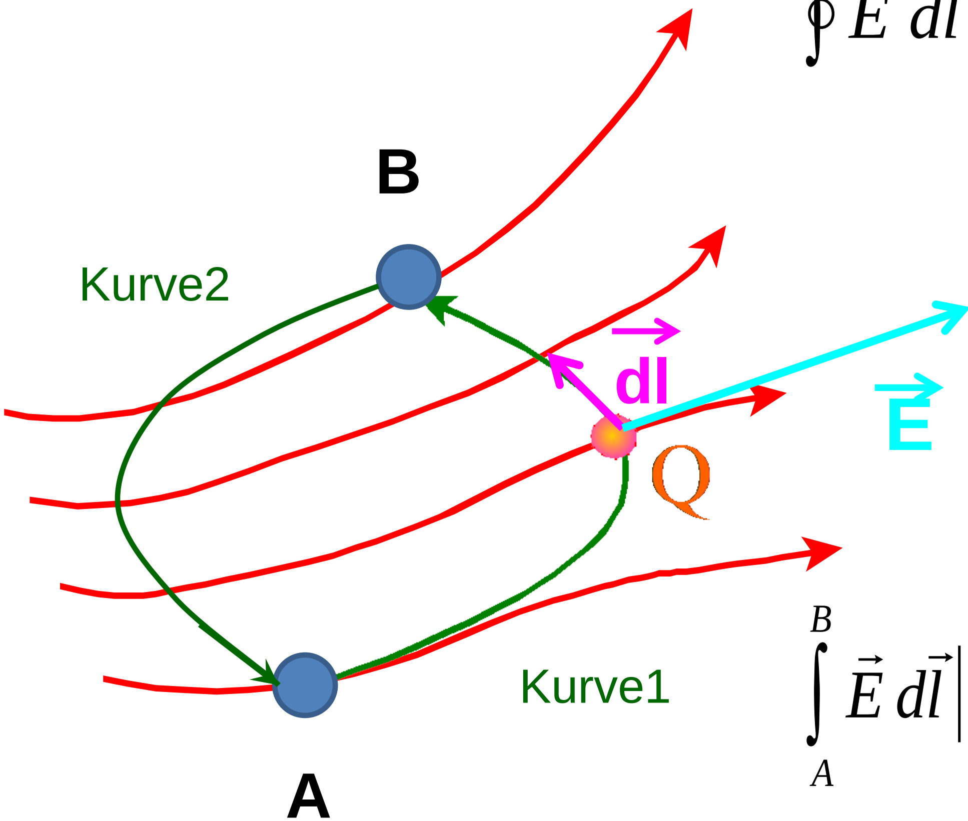
und

$$\boxed{\oint \vec{E} \cdot d\vec{l} = 0}$$

$$\oint \vec{E}\, d\vec{l} = 0$$

Dann ist auch

$$\oint \vec{E}\, d\vec{l} = \int_A^B \vec{E}\, d\vec{l}\Big|_{Kurve1} + \int_B^A \vec{E}\, d\vec{l}\Big|_{Kurve2} = 0$$

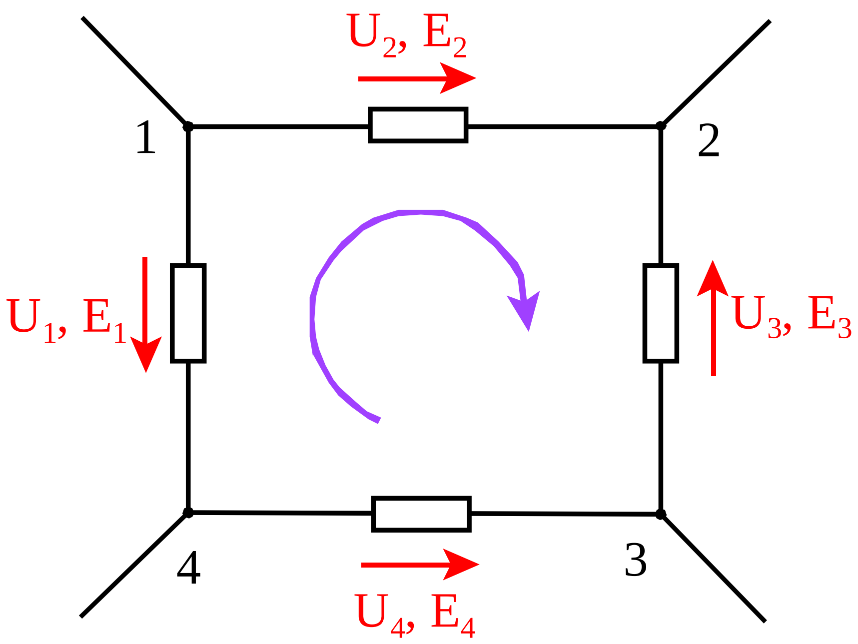


$$\int_A^B \vec{E}\, d\vec{l}\Big|_{Kurve1} = -\int_B^A \vec{E}\, d\vec{l}\Big|_{Kurve2} = \int_A^B \vec{E}\, d\vec{l}\Big|_{Kurve2}$$

und schließlich ergibt sich

$$U_{AB}\Big|_{Kurve1} = \int_A^B \vec{E}\, d\vec{l} = U_{AB}\Big|_{Kurve2}$$

Der Maschensatz (2. Kirchhoff´scher Satz)



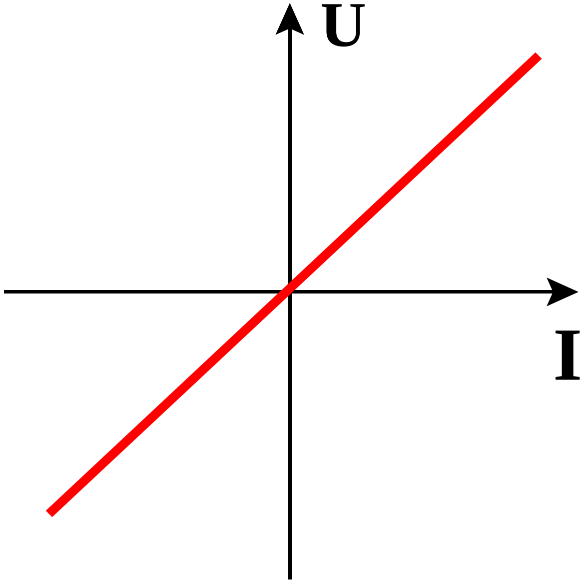
$$\oint \vec{E}\, d\vec{l} = 0$$

$$\int_1^2 \vec{E}\, d\vec{l} + \int_2^3 \vec{E}\, d\vec{l} + \int_3^4 \vec{E}\, d\vec{l} + \int_4^1 \vec{E}\, d\vec{l} = 0$$

$$+U_2 - U_3 - U_4 - U_1 = 0$$

für jede Masche gilt: $\sum_i U_{i\,vorz} = 0$ Maschensatz (2. Kirchhoff´scher Satz)

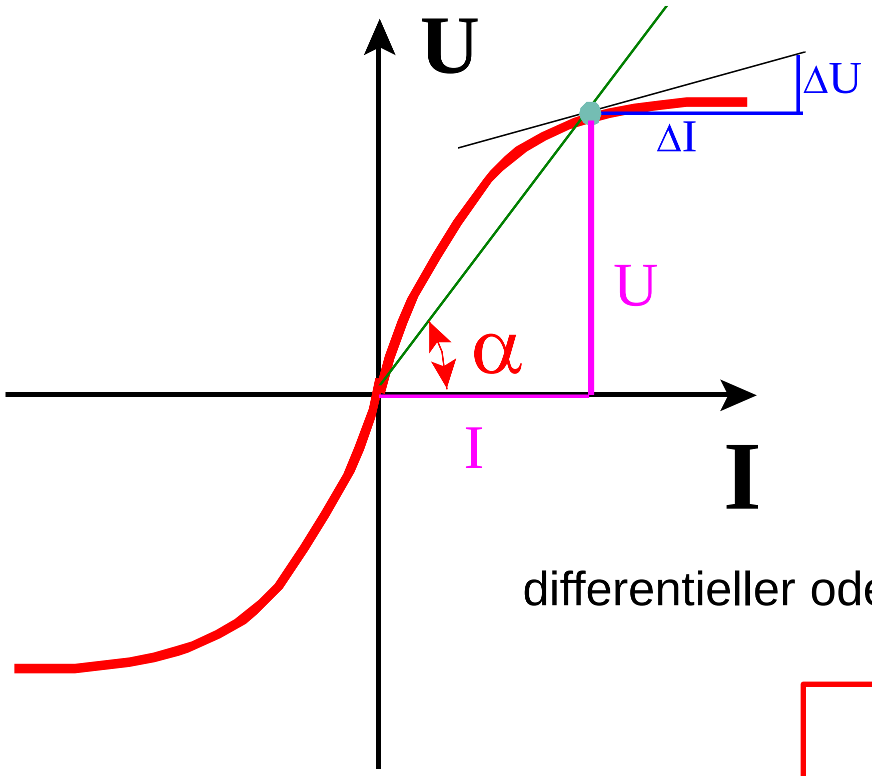
Zur Erinnerung: lineares passives Bauelement:



$$U \sim I$$

$$U = R\,I \qquad\qquad I = G\,U$$

nichtlineares Bauelement:

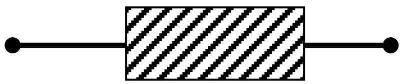


$$R = \frac{U}{I} = \tan\alpha$$

differentieller oder dynamischer Widerstand:

$$r_d = \lim_{\Delta I \to 0} \frac{\Delta U}{\Delta I} = \frac{dU}{dI}$$

$$r_d \neq R$$



R_2

$$U_2 = k_1 \cdot I^2 + k_2 \cdot I \quad mit$$

$$k_1 = 0{,}7 \frac{V}{A^2} \; und \; k_2 = 2 \frac{V}{A}$$

Statischer Widerstand R_s:

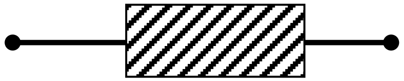
$$U_2 = 0{,}7 \frac{V}{A^2} \cdot I^2 + 2 \frac{V}{A} \cdot I$$

I in A	U2 in V	Rs in Ω
0	0	
2	6,8	3,4
4	19,2	4,8
6	37,2	6,2
8	60,8	7,6
10	90	9

differentieller Widerstand R_d:

$$\frac{dU_2}{dI} = 0{,}7 \cdot 2 \cdot I + 2$$

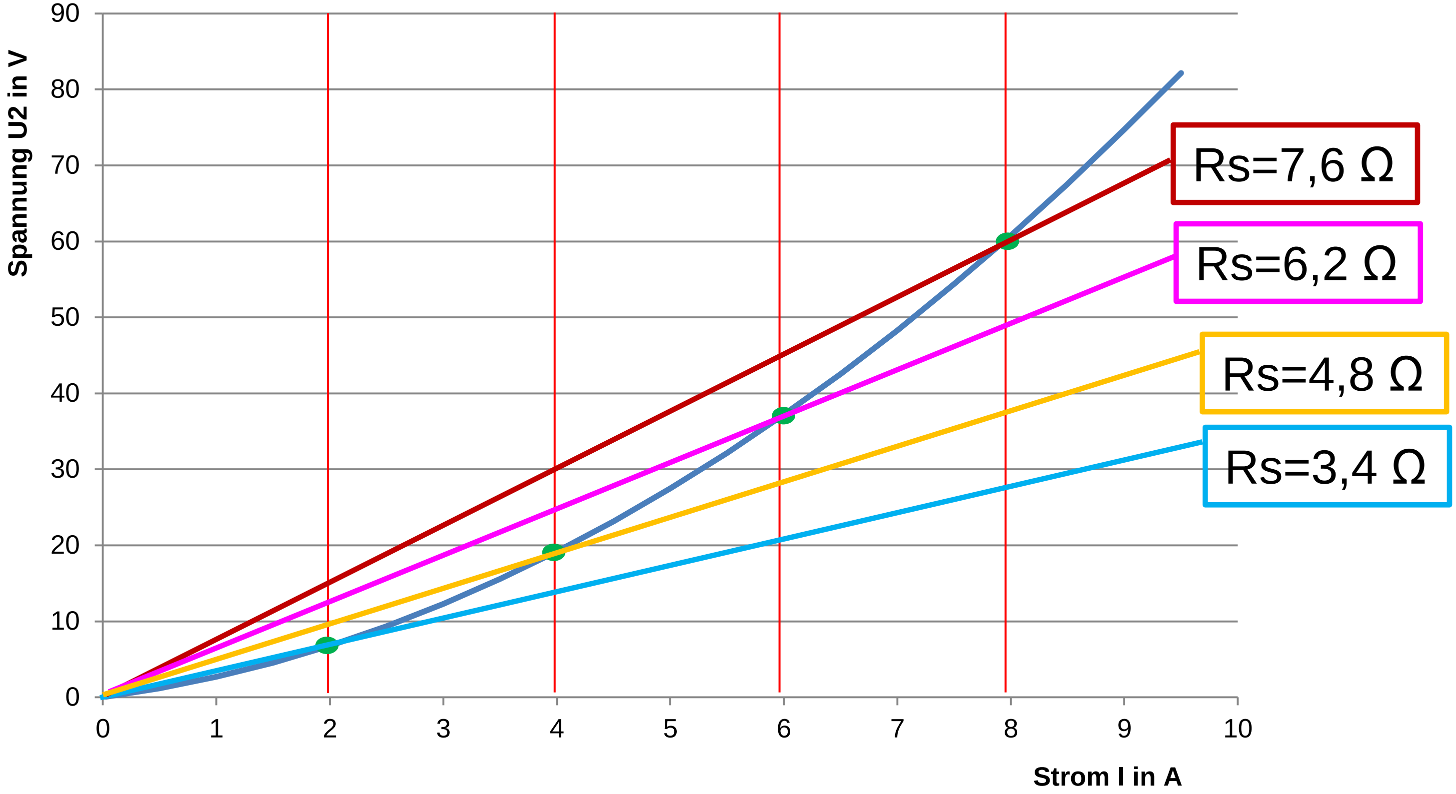
I in A	Rs in Ω
0	2
2	4,8
4	7,6
6	10,4
8	13,2
10	16

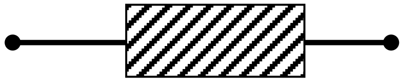


R_2

$$U_2 = 0,7 \cdot I^2 + 2 \cdot I$$

Statischer Widerstand R_s:

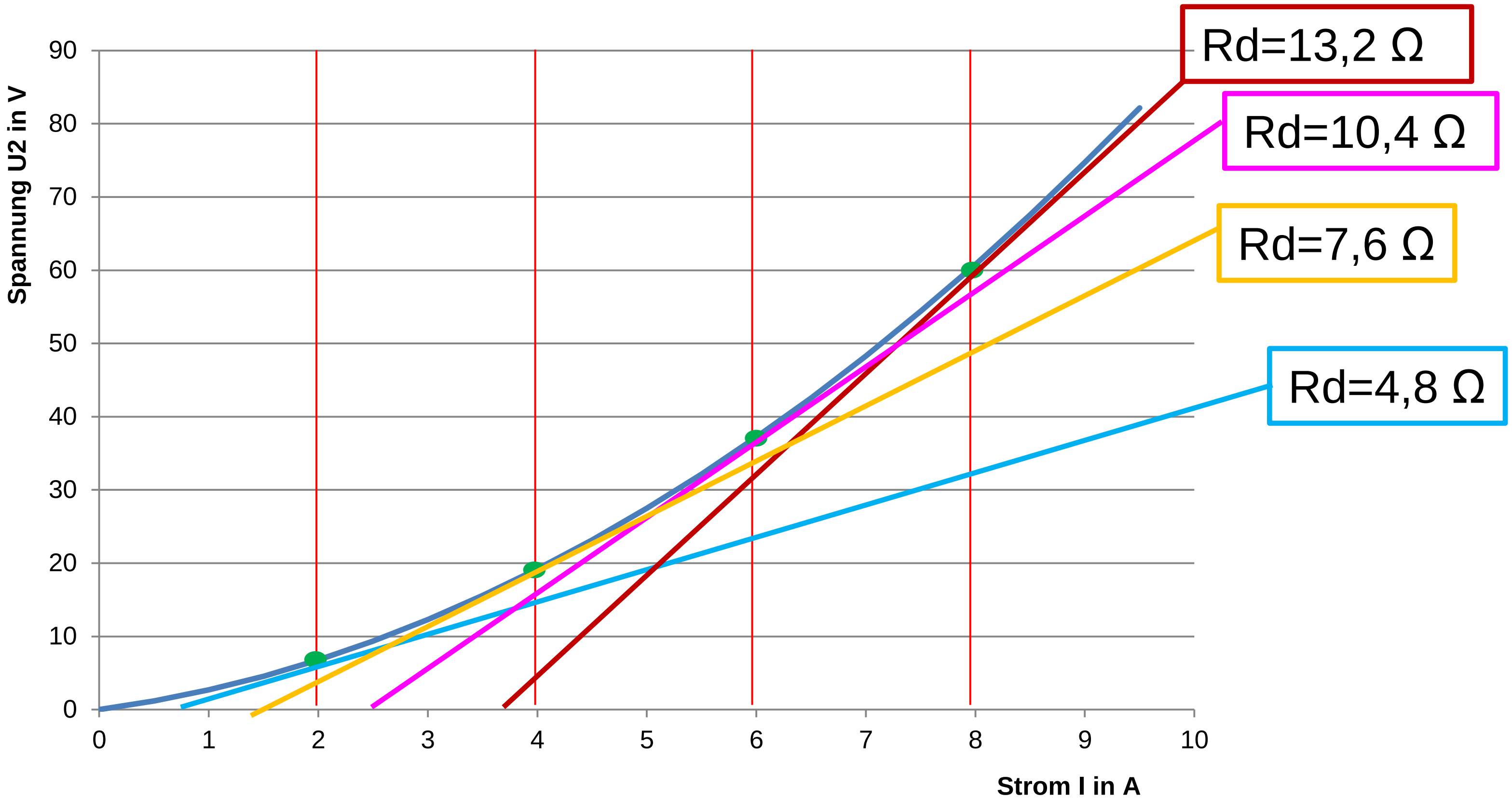




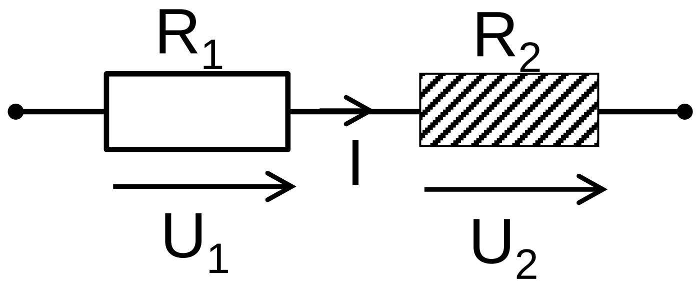
R_2

$$U_2 = 0{,}7 \cdot I^2 + 2 \cdot I$$

differentieller Widerstand R_s:



Reihenschaltung mit nichtlinearem Widerstand

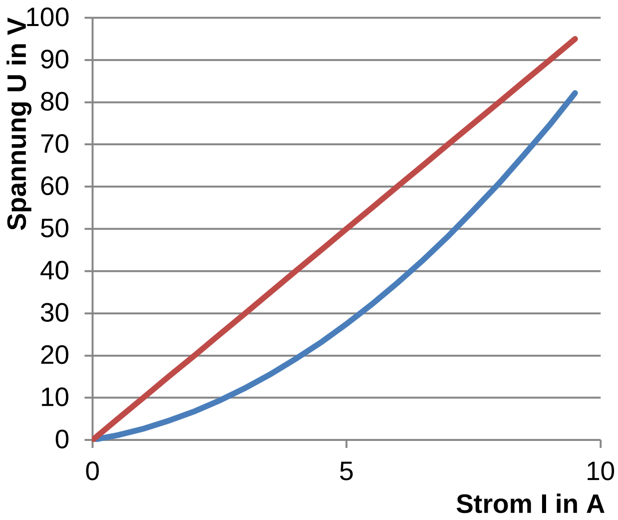


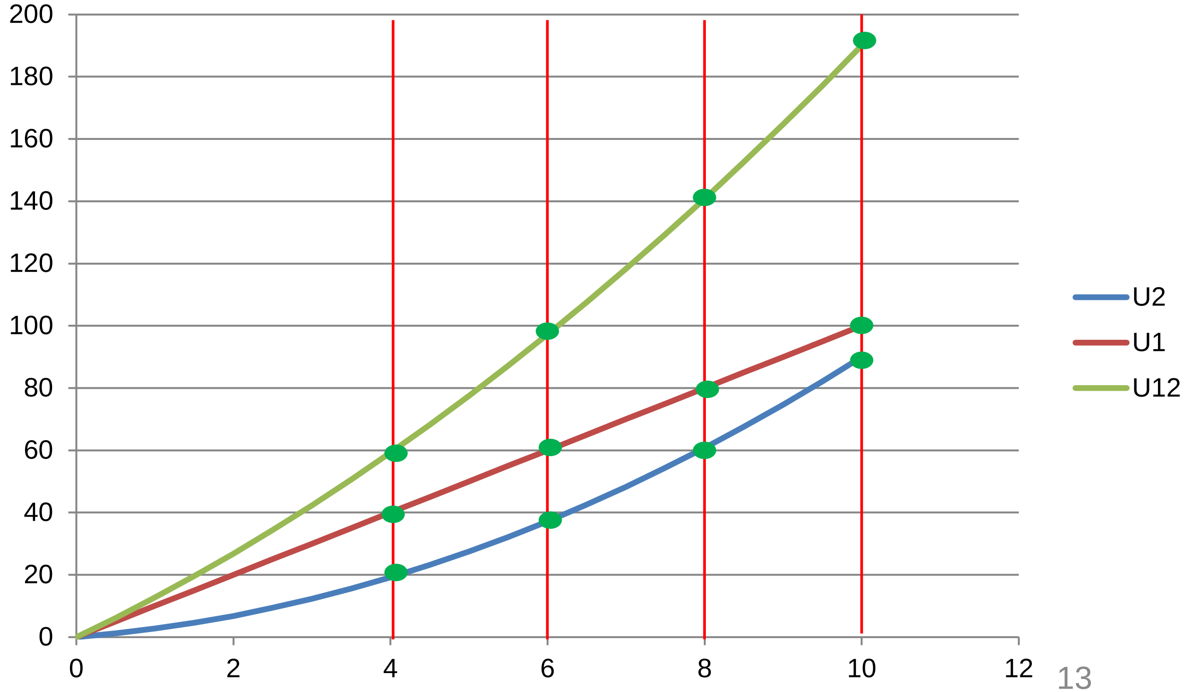
$$U_2 = k_1 \cdot I^2 + k_2 \cdot I \quad mit$$

$$k_1 = 0{,}7 \frac{V}{A^2} \; und \; k_2 = 2 \frac{V}{A}$$

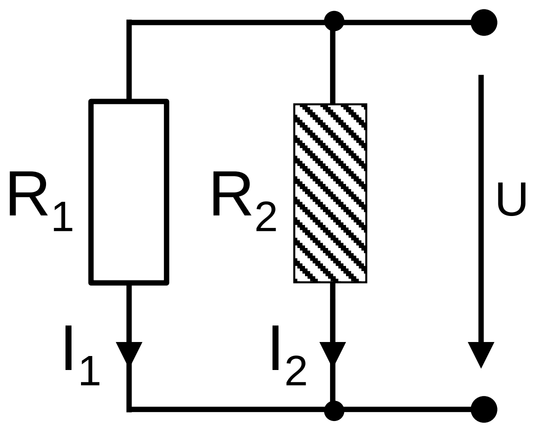
$R_1 = 10\Omega$

Bei gleichem Strom
Spannungen U1 und U2 addieren





Parallelschaltung mit nichtlinearem Widerstand

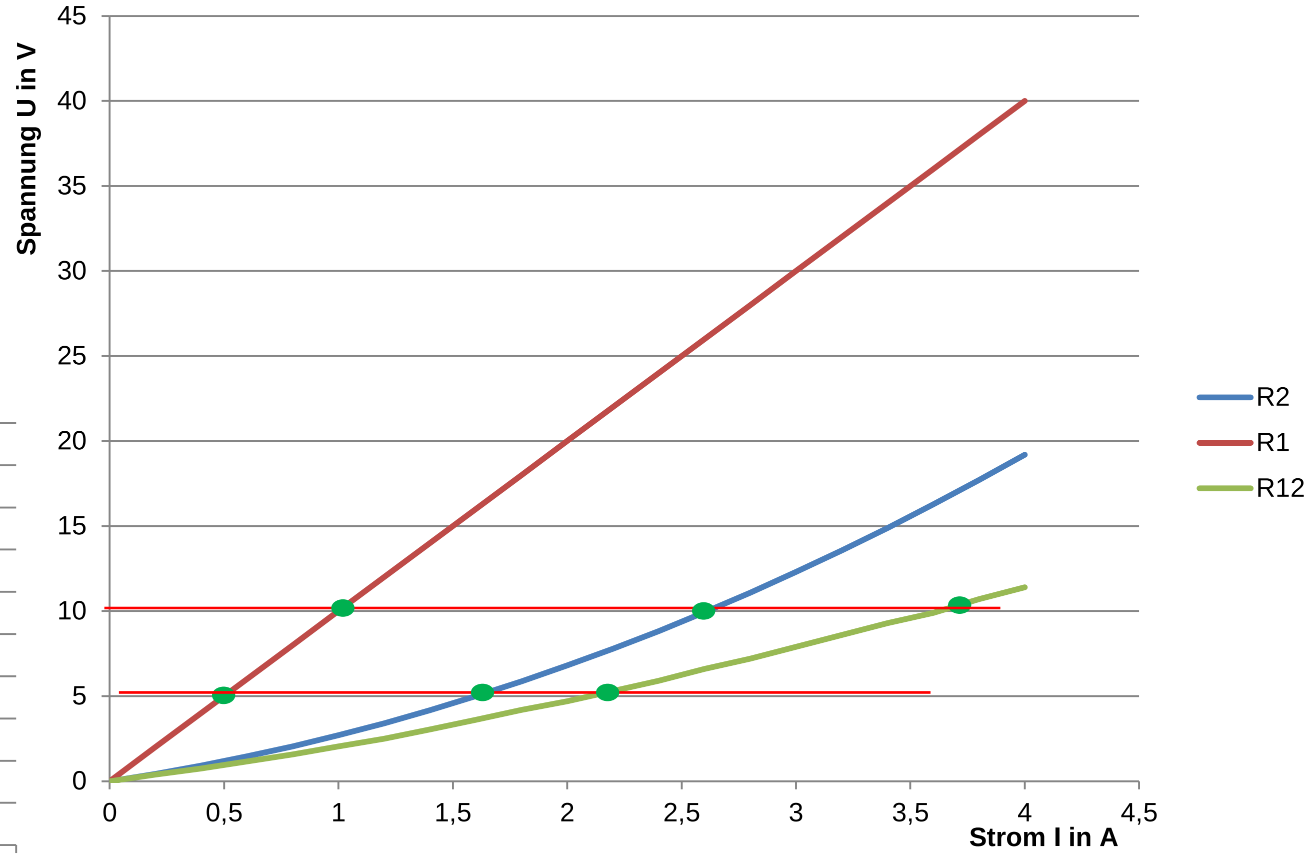


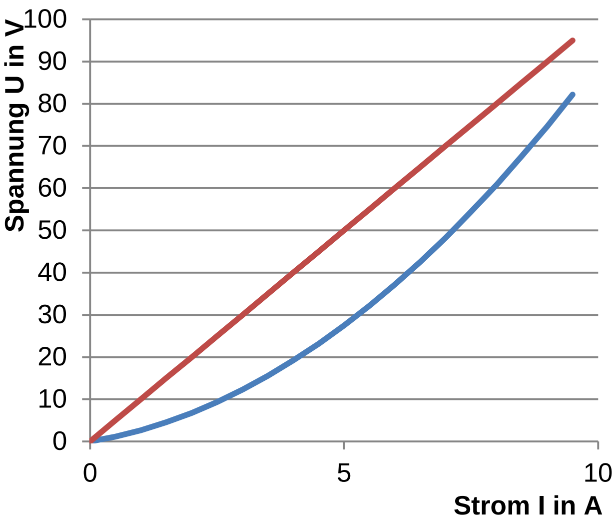
R_1=10Ω

$$U_2 = k_1 \cdot I^2 + k_2 \cdot I \quad mit$$

$$k_1 = 0{,}7\frac{V}{A^2} \; und \; k_2 = 2\frac{V}{A}$$

Bei gleicher Spannung Ströme I1 und I2 addieren





Materialgleichung des Strömungsfeldes

$$\vec{J} = \gamma * \vec{E}$$

$$[\gamma] = 1\frac{A}{Vm} = 1\Omega^{-1}m^{-1} = 1\frac{S}{m}$$

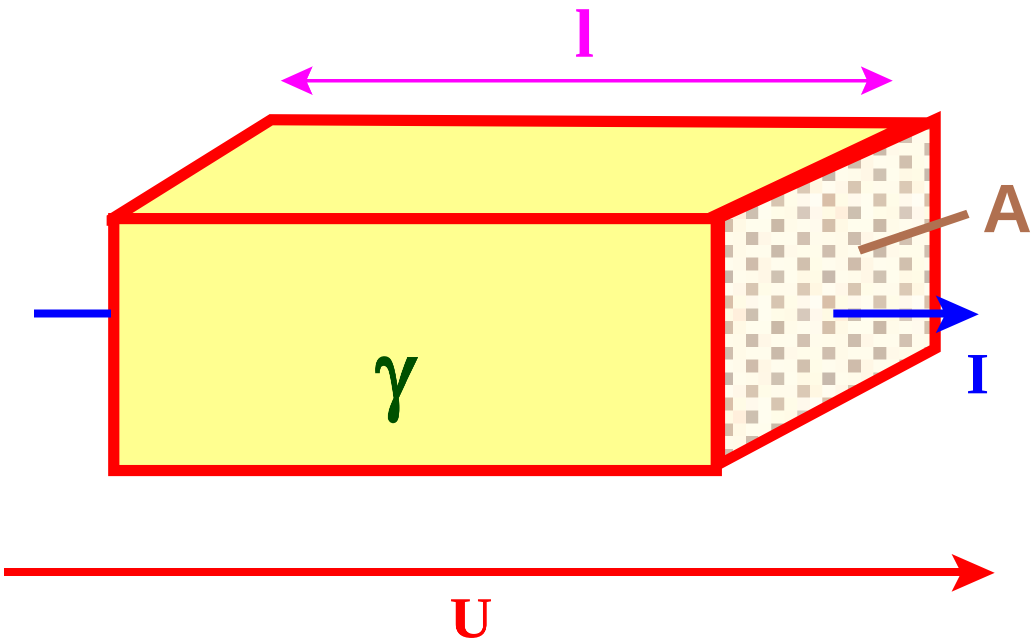
elektrische Leitfähigkeit, auch mit κ bezeichnet

der spezifische elektrische Widerstand ist dann:

$$\rho = \frac{1}{\gamma}$$

$$[\rho] = 1\Omega m = 1\frac{\Omega mm^2}{m} = 1\Omega cm$$

die Widerstandsbemessungsgleichung



$$J = \gamma E$$

im homogenen Feld gilt:

U = E * l

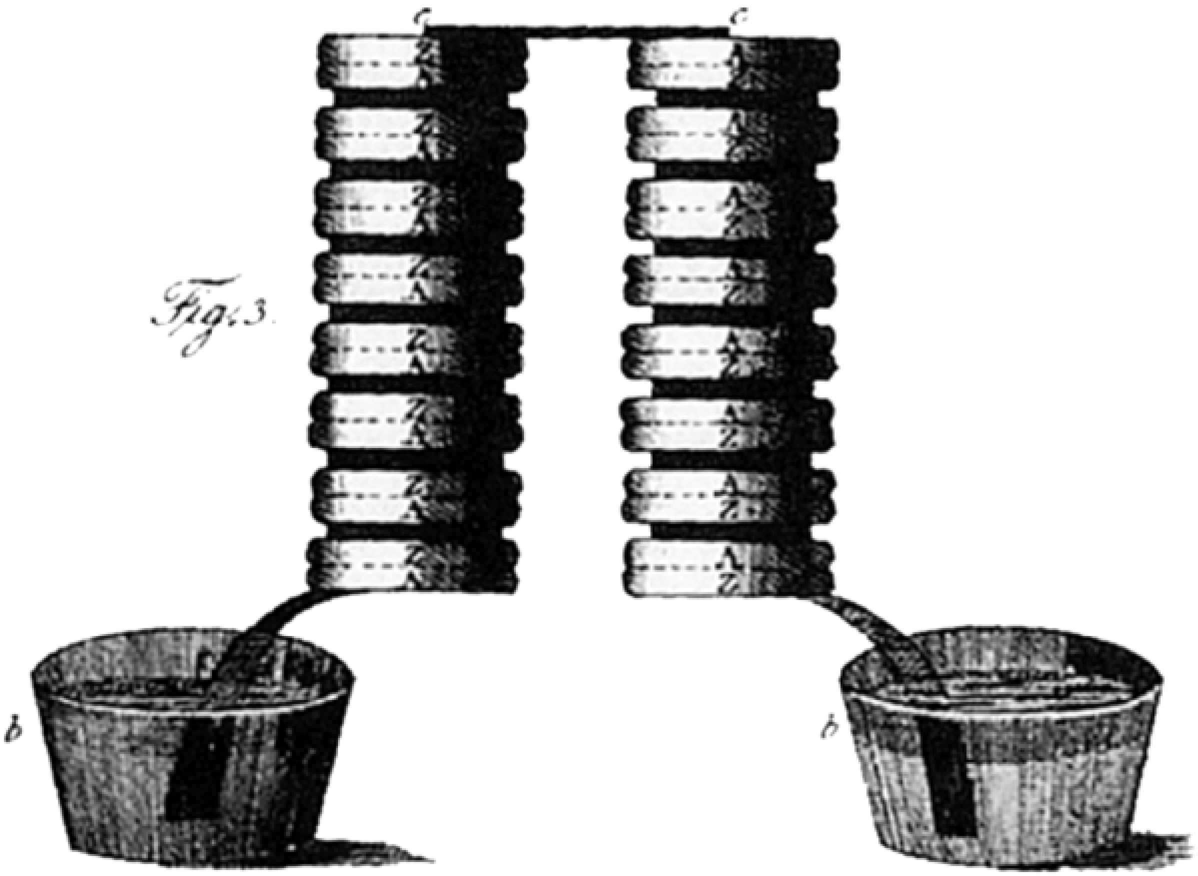
I = J * A

und damit wird für R:

$$R = \frac{U}{I} = \frac{E\,l}{J\,A} = \frac{E\,l}{\gamma\,E\,A}$$

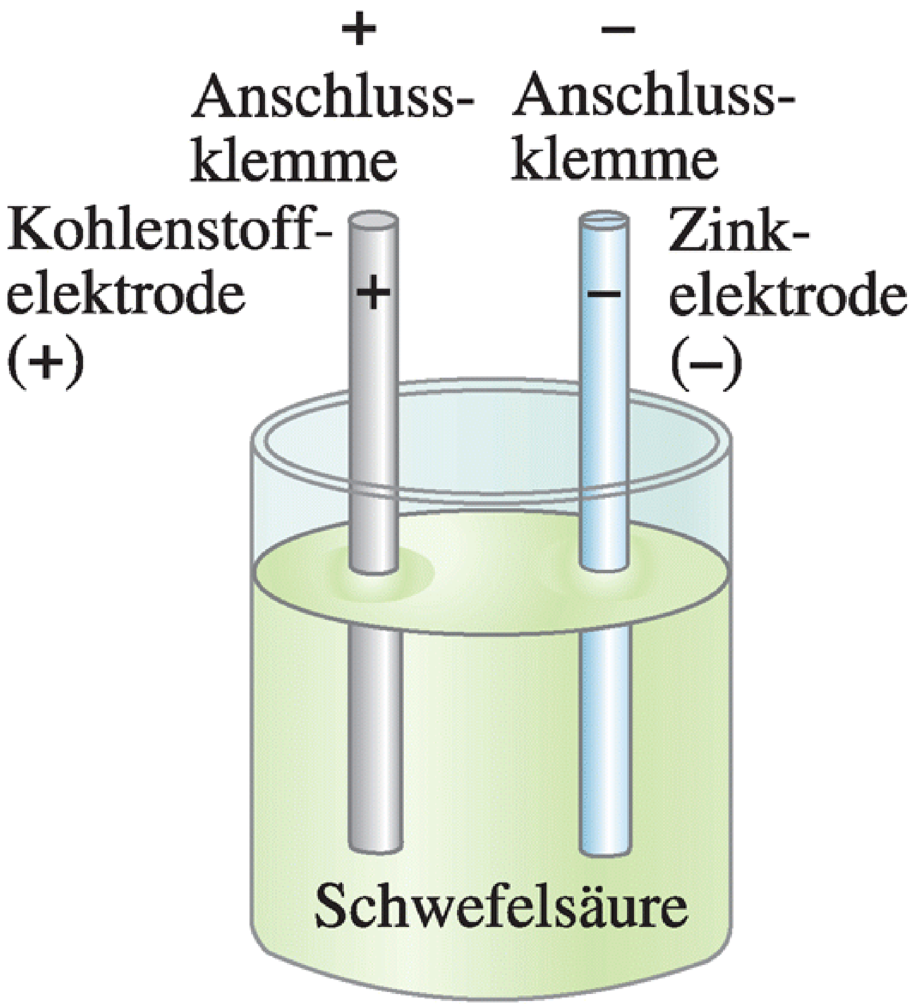
$$R = \frac{l}{\gamma\,A} = \rho\,\frac{l}{A}$$

2.4 aktive Elemente (Spannungs- und Stromquellen)



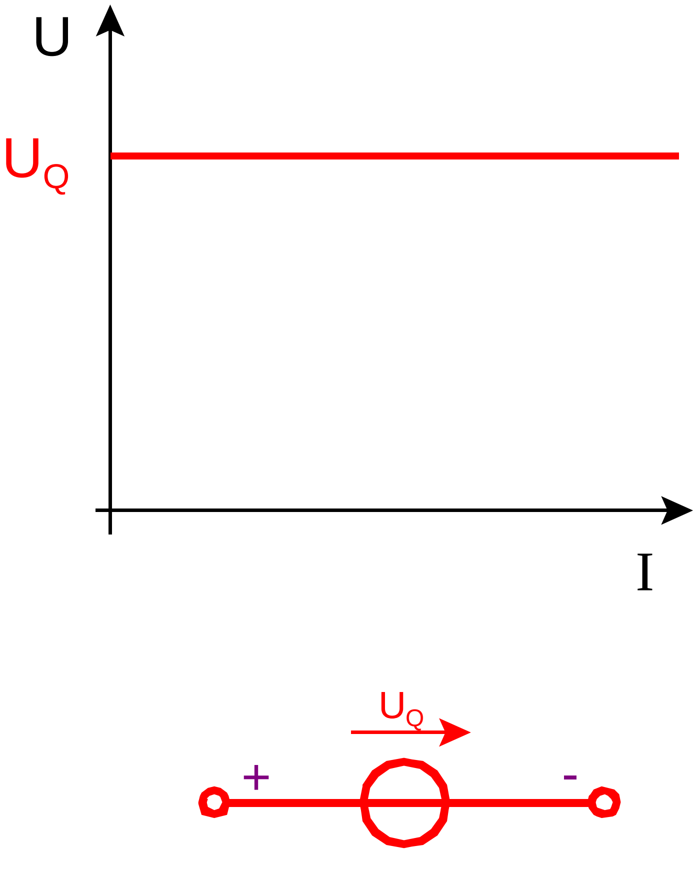
Eine voltaische Batterie, entnommen aus der Originalveröffentlichung von Volta

Ein einfaches galvanisches Element:

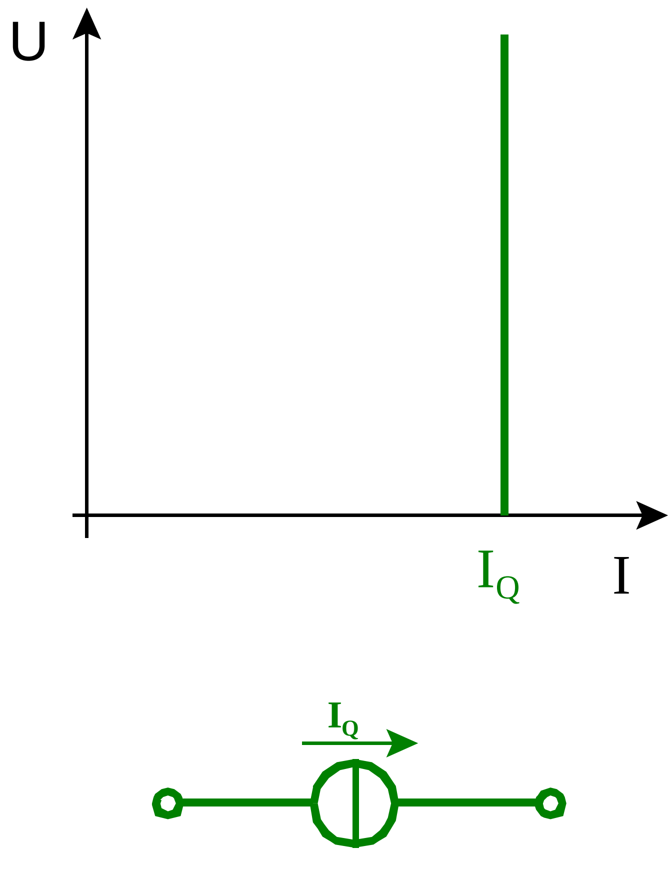


das ideale aktive Element

Modell der Spannungsquelle

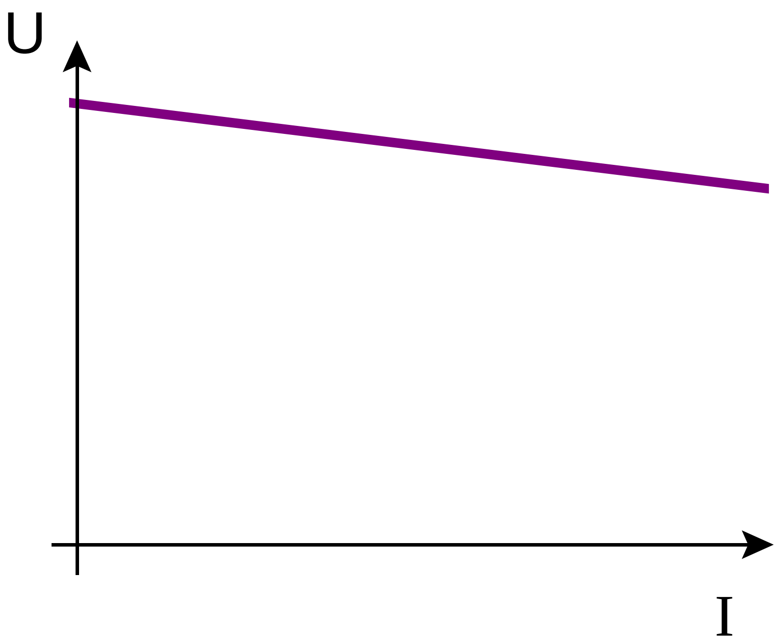


Modell der Stromquelle



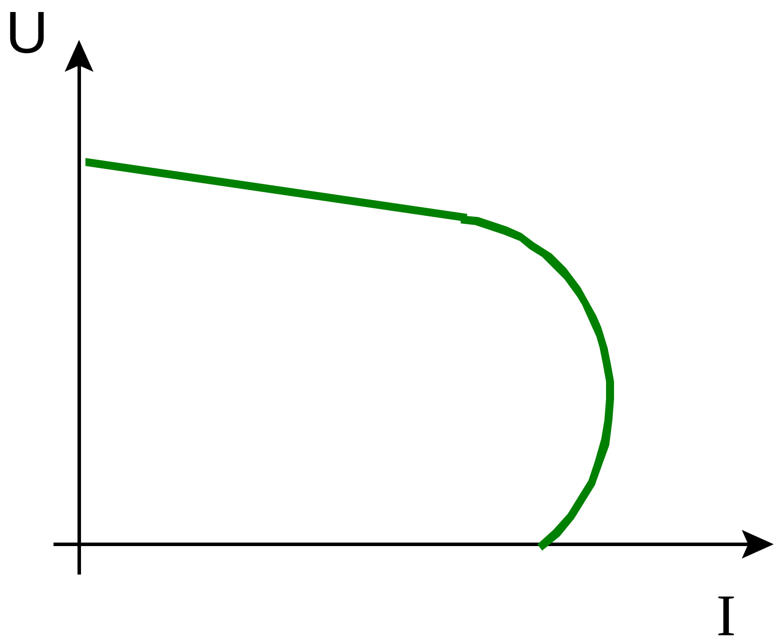
- die Strom-Spannungs-Kennlinie realer aktiver Elemente

linearer Fall



Akkumulator

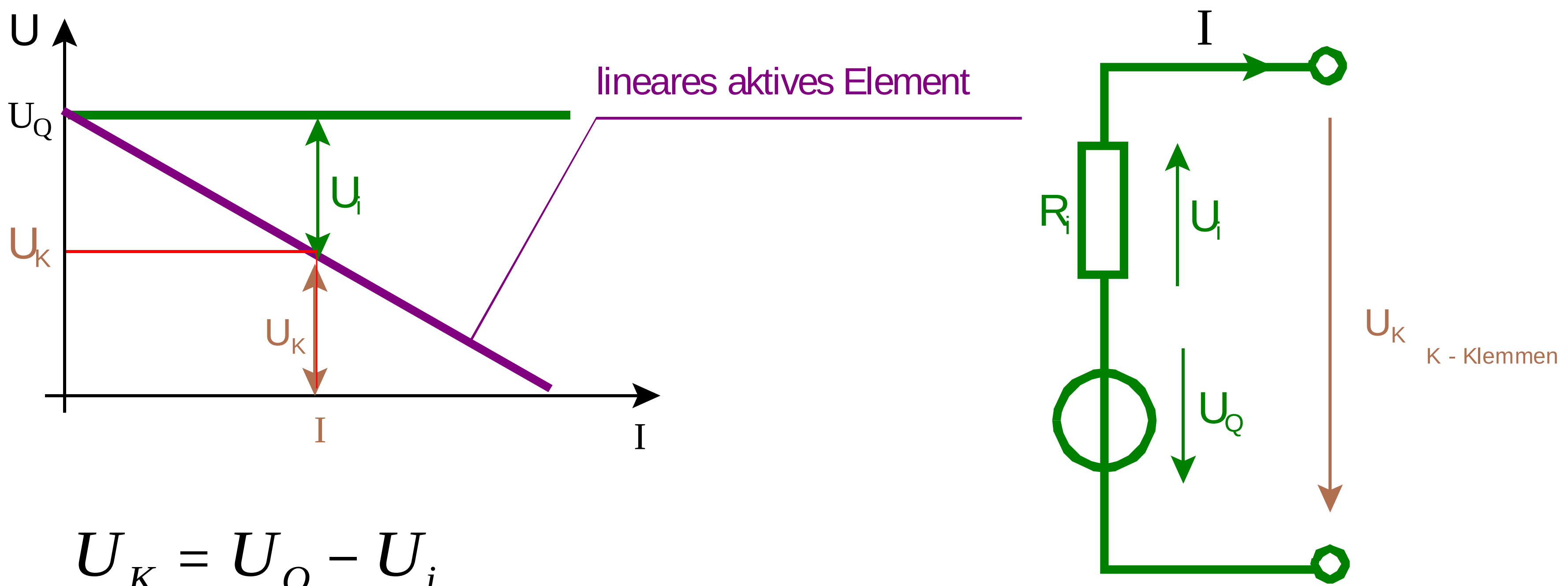
nichtlinearer Fall



Gleichstromnebenschlußgenerator

2.4.2 Modellierung des elektrischen Verhaltens eines realen linearen aktiven Elementes durch Ersatzschaltbilder

a) das Spannungsquellenersatzschaltbild

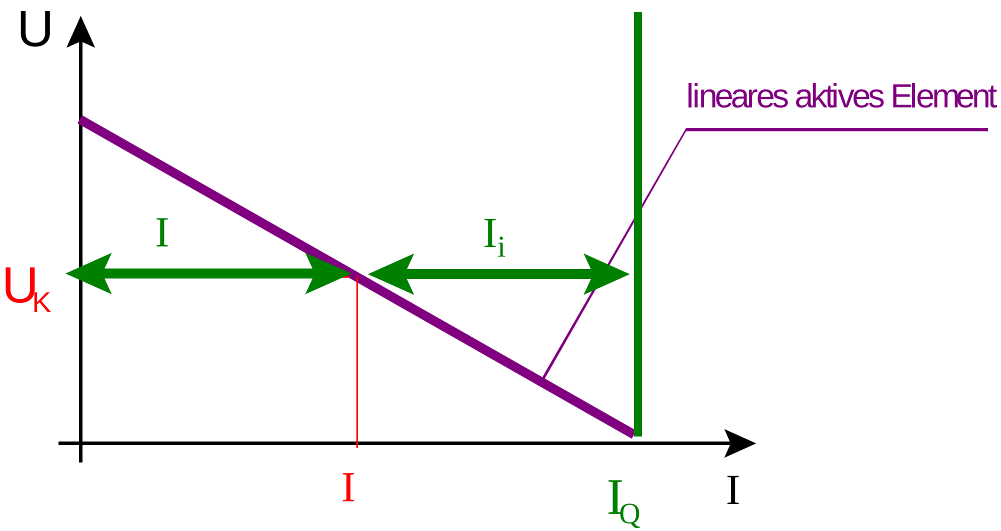


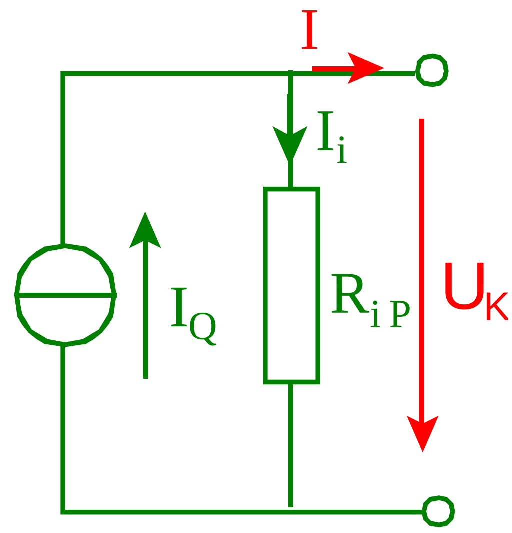
$$U_K = U_Q - U_i$$

$$U_i = R_i \, I$$

$$U_K = U_Q - R_i \, I$$

b) das Stromquellenersatzschaltbild





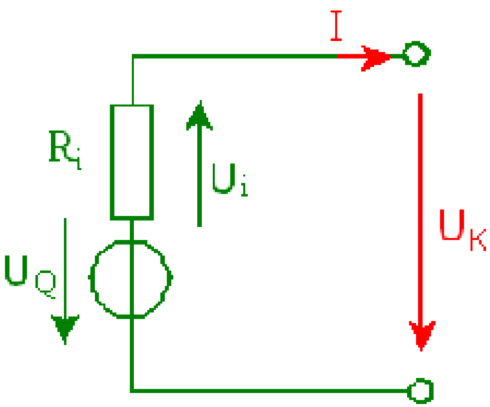
$$I = I_Q - I_i$$

$$I_i = \frac{U_K}{R_{i\,P}}$$

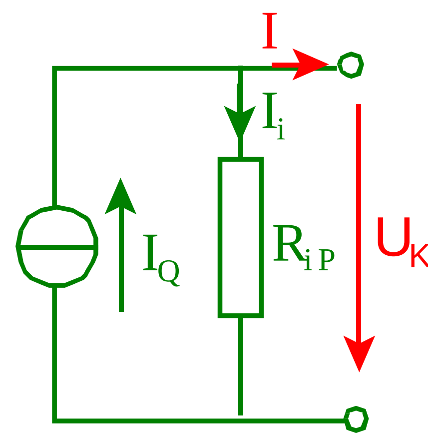
$$\boxed{I = I_Q - \frac{U_K}{R_{i\,P}}}$$

c) Äquivalenz von Spannungs- und Stromquellenersatzschaltbild

Spannungsquelle:



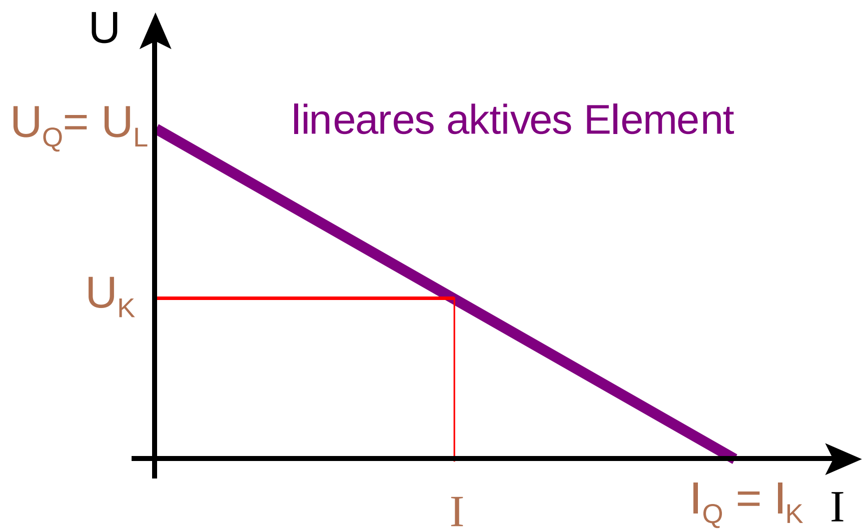
Stromquelle:



$$R_i = R_{i\,P}$$

$$U_Q = I_Q \, R_i$$

$$U_K = U_Q - R_i \, I$$



$$I = I_Q - \frac{U_K}{R_{i\,P}}$$

d) die extremen Betriebszustände aktiver Elemente

$$U_K = U_Q - R_i\, I$$

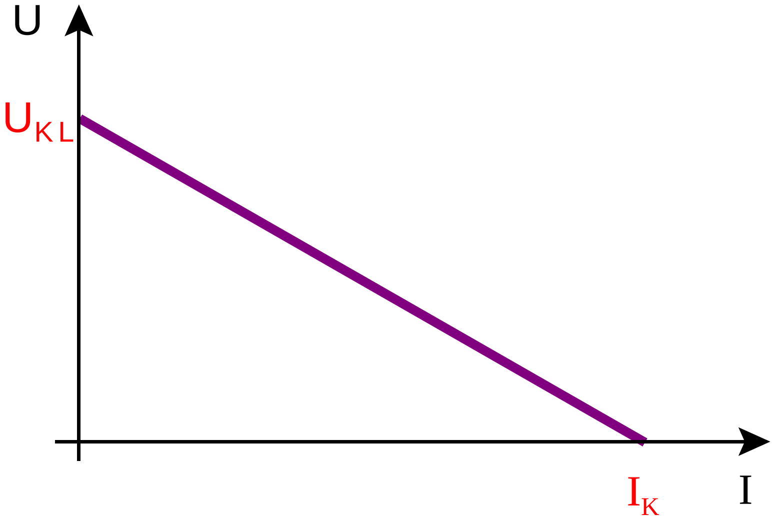
$$I = I_Q - \frac{U_K}{R_{i\,P}}$$

1. Leerlauf (I=0):

$$U_{K\,L} = U_Q$$

2. Kurzschluss (U_{Kl}=0):

$$I_K = I_Q = \frac{U_Q}{R_i}$$



2.4.3 elektrische Leistung und Wirkungsgrad

$$W_{el} = Q\,U \qquad dW_{el} = dQ\,U$$

$$P_{el} = \frac{dW_{el}}{dt} = \frac{dQ}{dt}U$$

elektrische Leistung:

$$\boxed{P_{el} = U\,I}$$

mit der Einheit [P]=[U] [I]=1VA=1W

- der Satz von der Erhaltung der Leistung

$$\sum_i W_i = konst.$$

$$\frac{d}{dt}\sum_i W_i = \sum_i \frac{dW_i}{dt} = 0$$

$$\sum_i P_i = 0$$

- der Wirkungsgrad

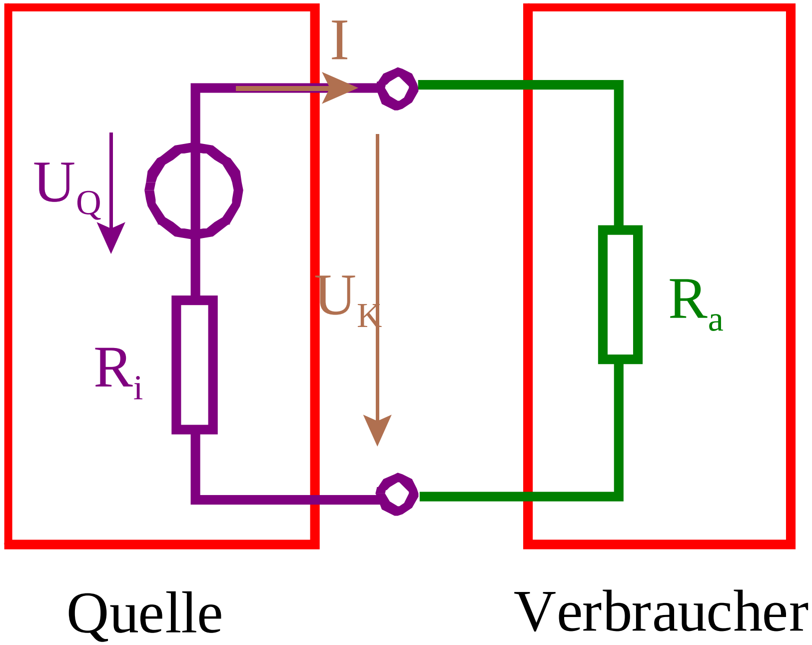
$$\eta = \frac{W_{ab}}{W_{zu}}$$

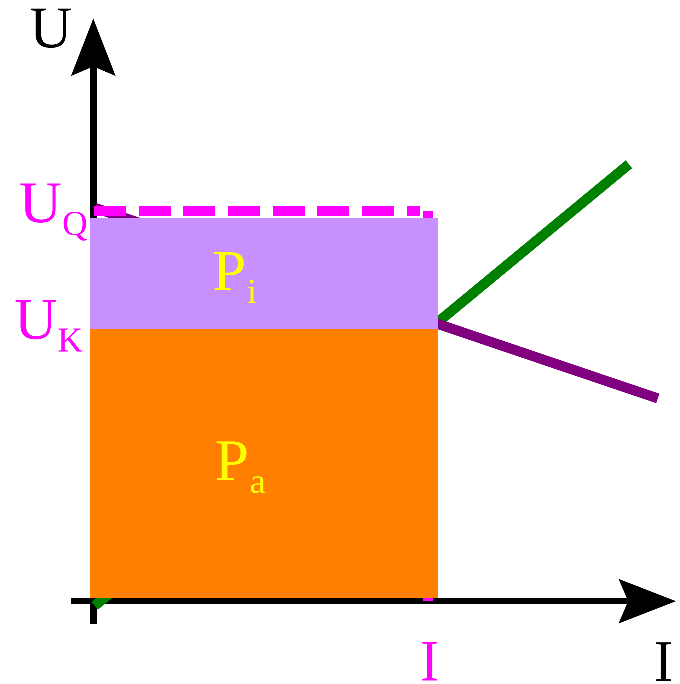
ohne Energiespeicher

$$\eta = \frac{P_{ab}}{P_{zu}}$$

2. 5 Der Grundstromkreis

2.5.1 lineare Quelle und linearer Verbraucher

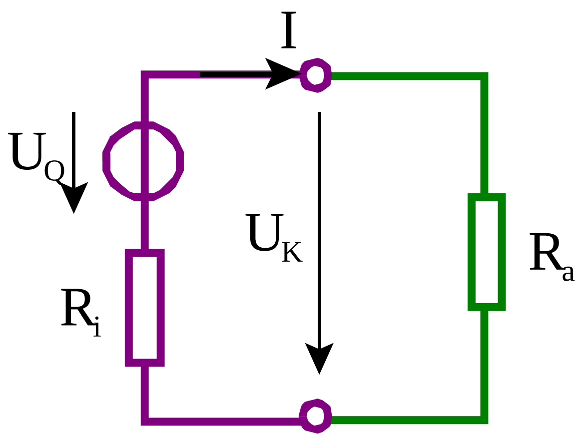




$$I_{Quelle} = I_{Verbraucher}$$

$$U_{K\,Quelle} = U_{K\,Verbraucher}$$

- rechnerische Lösung



$$U_{K\,Quelle} = U_Q - R_i\, I = U_{K\,Verbraucher} = I\, R_a$$

$$I = \frac{U_Q}{(R_i + R_a)}$$

$$U_K = \frac{R_a\, U_Q}{(R_i + R_a)}$$

$$P_a = I\, U_K = \frac{R_a\, U_Q^{\,2}}{(R_i + R_a)^2}$$

$$\eta = \frac{P_a}{P_Q} = \frac{U_K I}{U_Q I} = \frac{R_a}{R_i + R_a}$$

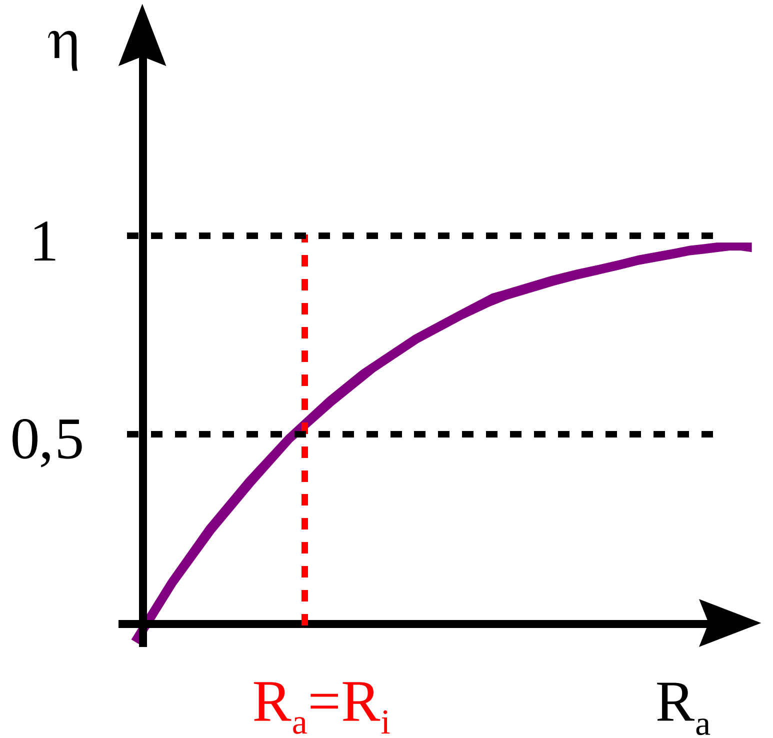
- Kurvendiskussion

$$\eta = \frac{P_a}{P_Q} = \frac{U_K I}{U_Q I} = \frac{R_a}{R_i + R_a}$$

$$\eta(R_a = 0) = 0$$

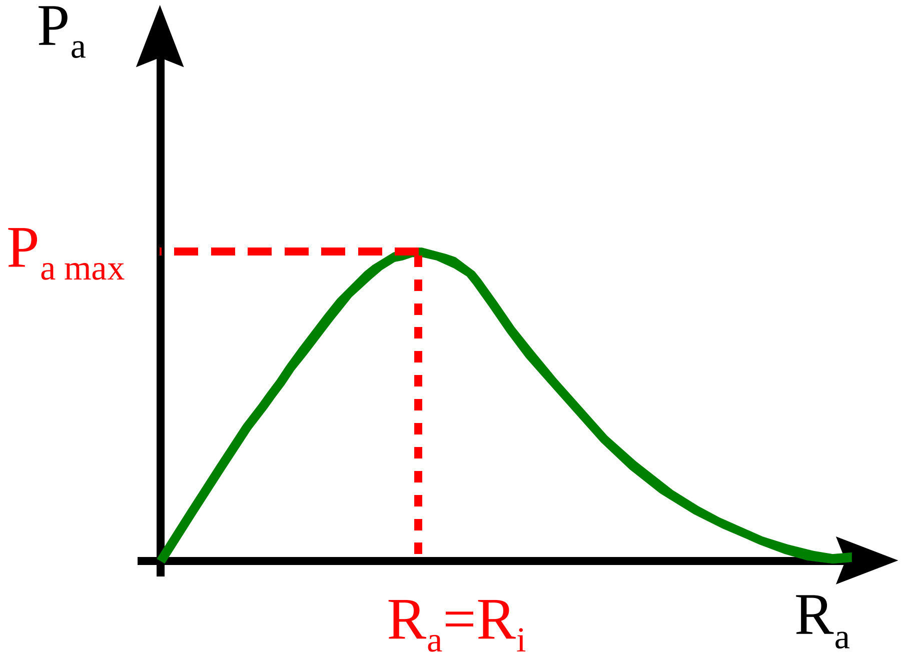
$$\eta\,(R_a = R_i) = 0{,}5$$

$$\eta(R_a \to \infty) = 1$$



- **Leistung im Grundstromkreis**

$$P_a = I\,U_K = \frac{R_a\,U_Q^{\,2}}{(R_i + R_a)^2}$$



$$P_a(R_a = 0) = 0$$

$$P_a(R_a \to \infty) = 0$$

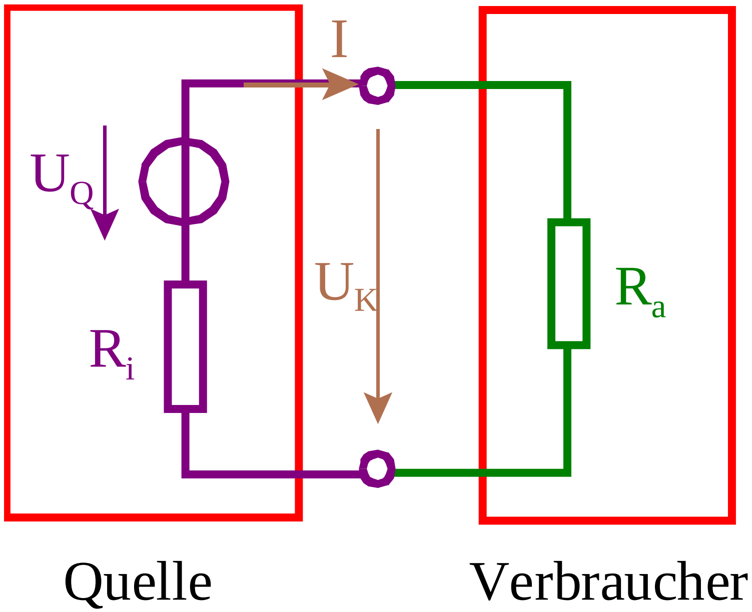
(u´v-uv´)/v^2

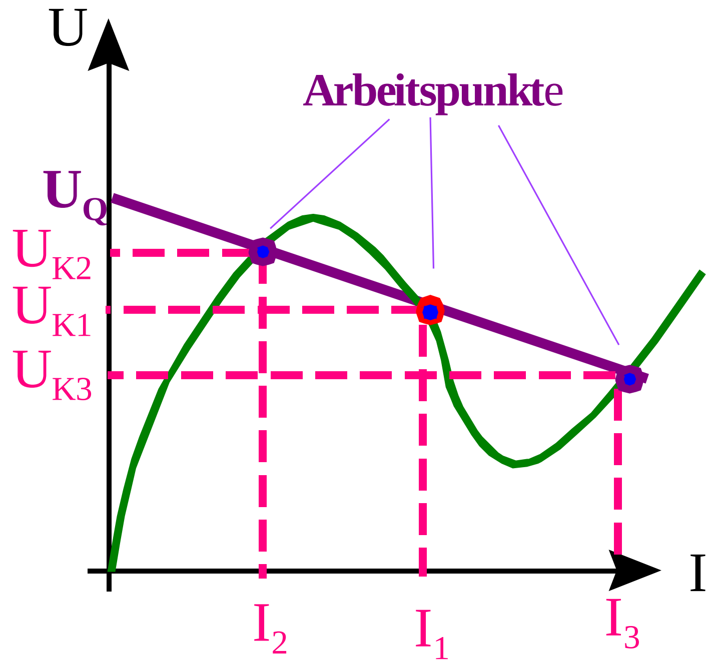
$$\frac{dP_a}{dR_a} = \frac{(R_i + R_a)^2 - 2R_a(R_i + R_a)}{(R_i + R_a)^4}U_Q^2 = 0$$

Leistungsanpassung

$$(R_i + R_a) - 2R_a = 0$$

$$R_a = R_i$$





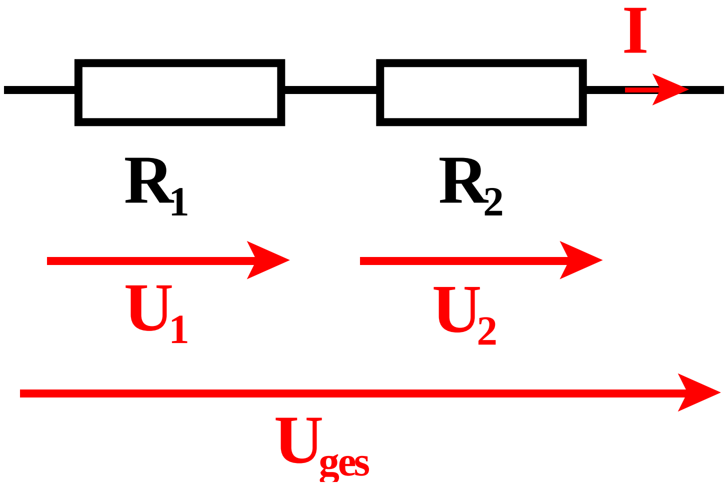
$$I_{Quelle} = I_{Verbraucher}$$

$$U_{K\,Quelle} = U_{K\,Verbraucher}$$

2.6 Berechnungsmethoden elektrischer Gleichstromkreise

2.6.1 Spannungs- und Stromteiler

a) Spannungsteilerregel



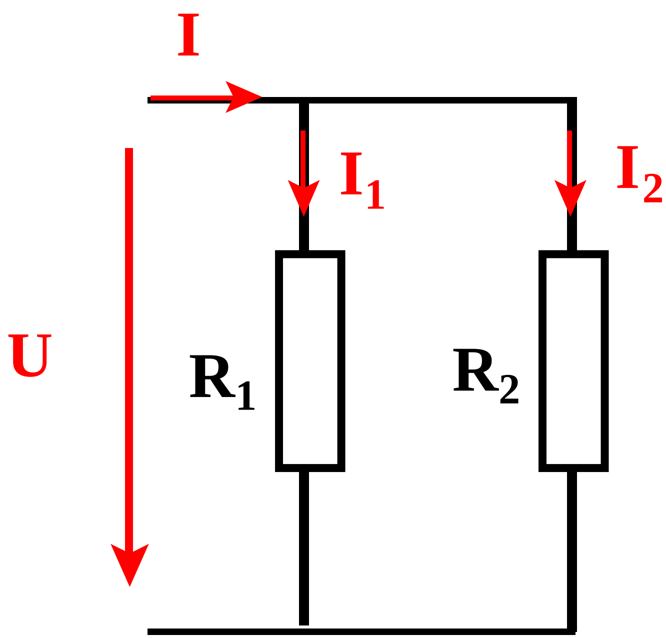
$$\frac{U_1}{U_2} = \frac{R_1}{R_2}$$

$$\frac{U_1}{U_{ges}} = \frac{R_1}{R_1 + R_2}$$

$$I_1 = I_2 = I$$

$$I_1 = \frac{U_1}{R_1} = I_2 = \frac{U_2}{R_2} = I = \frac{U_{ges}}{R_1 + R_2}$$

b) Stromteilerregel



$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\frac{I_1}{I} = \frac{R_{ersP}}{R_1} = \frac{R_1\,R_2}{R_1\left(R_1 + R_2\right)}$$

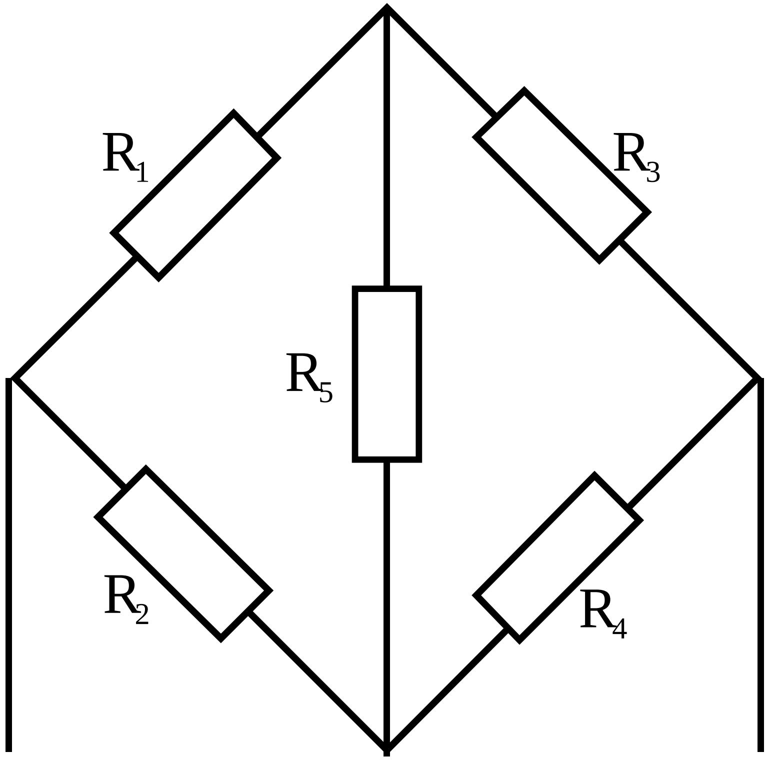
$$\boxed{\frac{I_1}{I} = \frac{R_{ersP}}{R_1} = \frac{R_2}{R_1 + R_2}}$$

$$U_1 = U_2 = U$$

$$U_1 = I_1\,R_1 = U_2 = I_2\,R_2 = U = I\,R_{ersP}$$

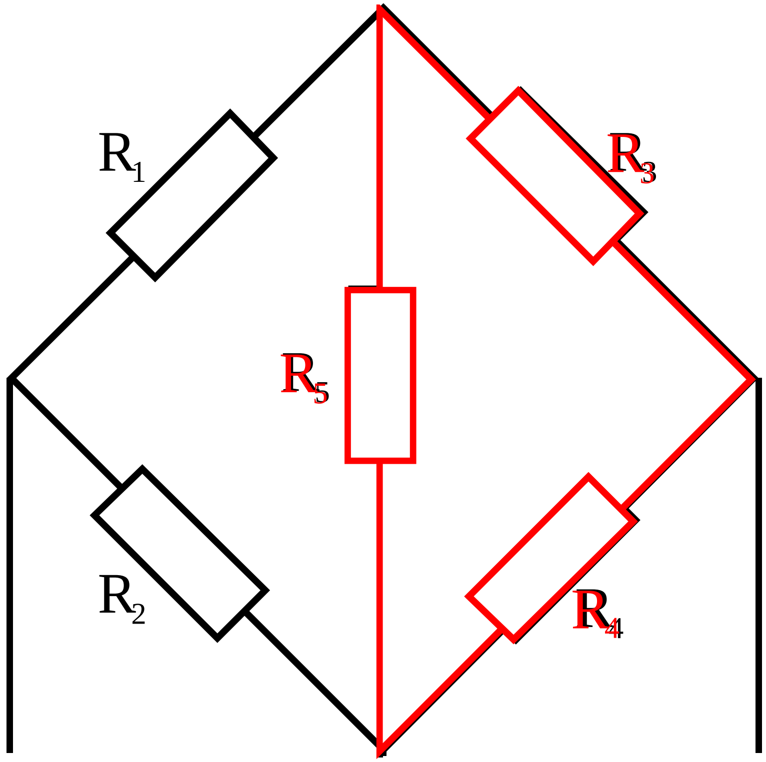
2.6.2 Brückenschaltung

Aufgabe: Bestimmen Sie den Ersatzwiderstand der folgenden Schaltung!

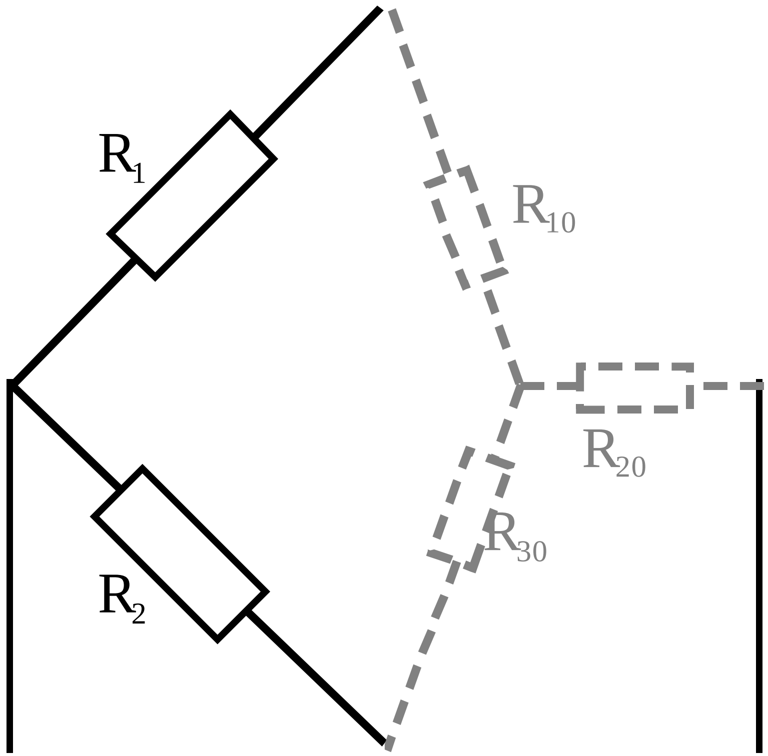


Problem: Lösung mit bisherigen Methoden nicht möglich!

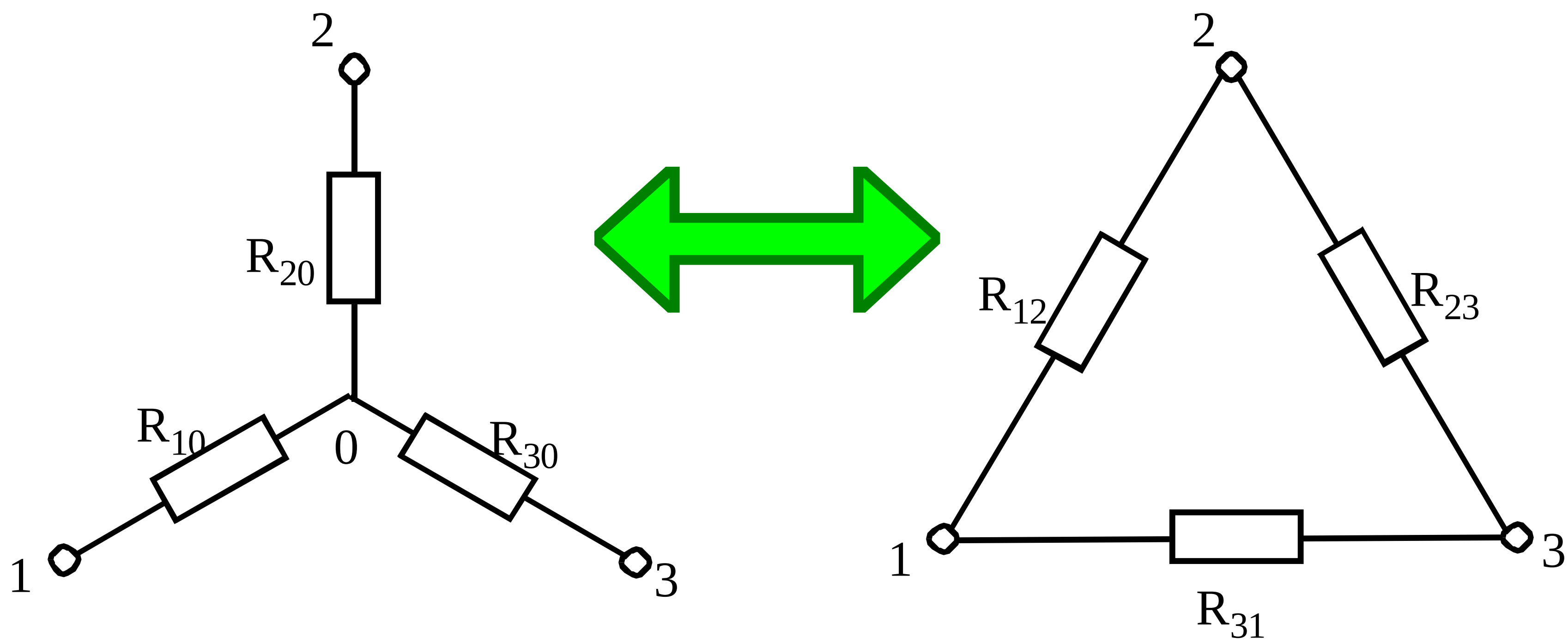
Ausweg: Umwandlung eines Widerstandsdreiecks in einen elektrisch äquivalenten Stern oder umgekehrt



Ausweg: Umwandlung eines Widerstandsdreiecks in einen elektrisch äquivalenten Stern oder umgekehrt



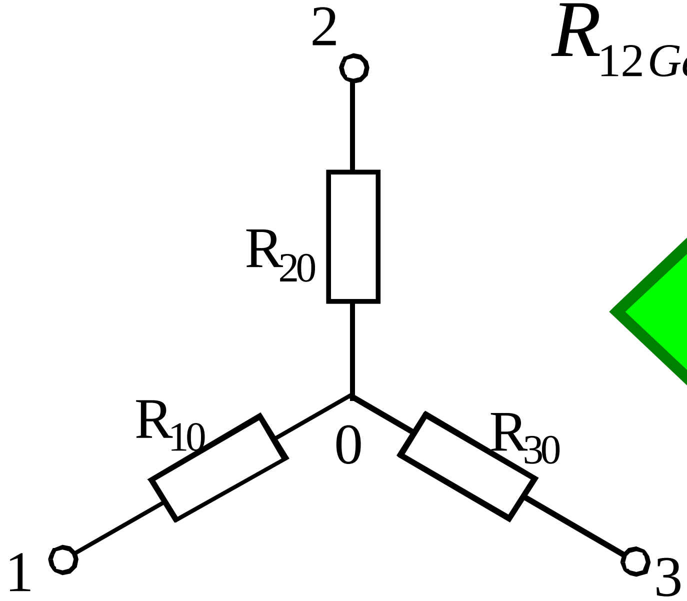
- Stern - Dreieck -Transformation:

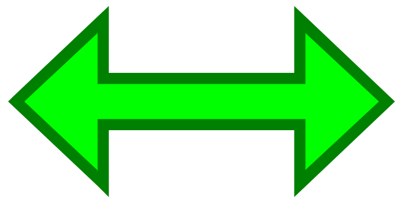


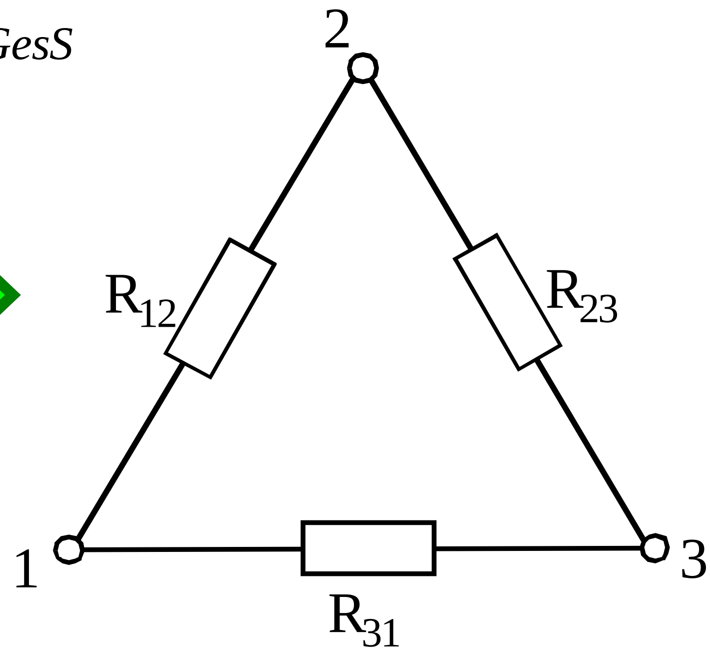
Elektrische Äquivalenz von Stern und Dreieck liegt vor, wenn ...

$$R_{12GesD} = R_{12GesS} \qquad R_{23GesD} = R_{23GesS} \qquad R_{31GesD} = R_{31GesS}$$

$$R_{12GesD} = R_{12GesS}$$





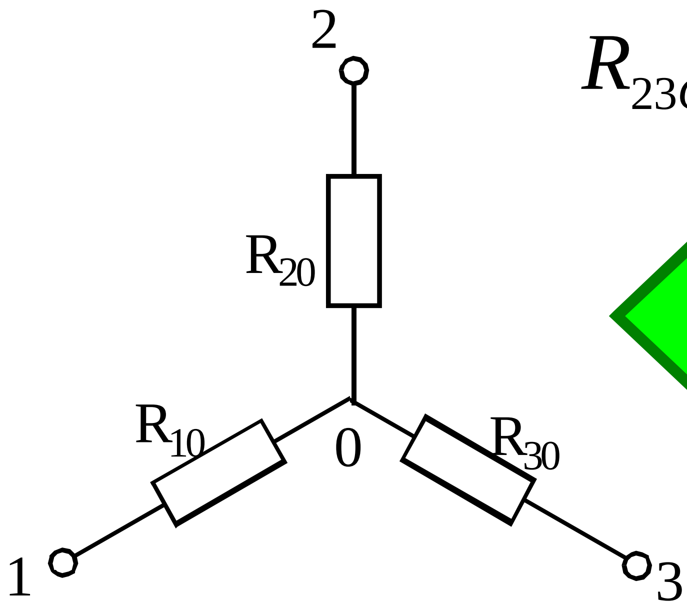


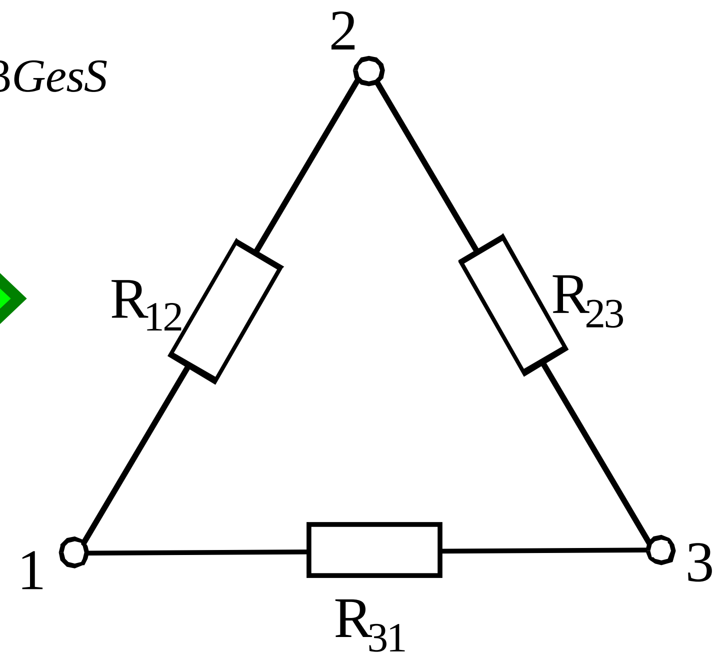
$$R_{12GesS} = R_{10} + R_{20}$$

$$R_{12GesD} = \frac{R_{12}\left(R_{23} + R_{31}\right)}{R_{12} + R_{23} + R_{31}}$$

$$R_{12GesD} = \frac{R_{12}R_{23} + R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{12GesS} = R_{10} + R_{20}$$

$$R_{23GesD} = R_{23GesS}$$



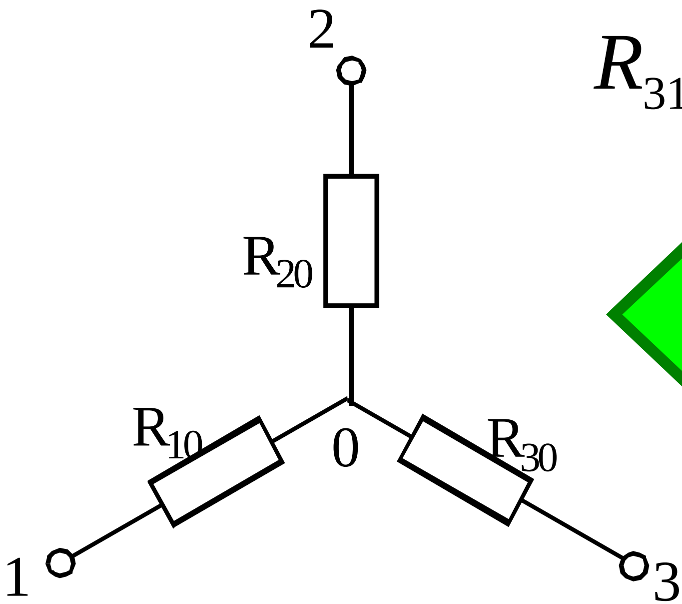


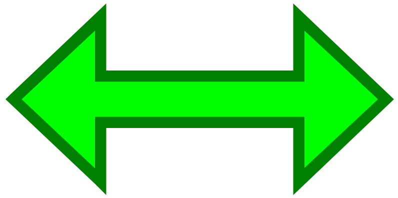
$$R_{23GesS} = R_{20} + R_{30}$$

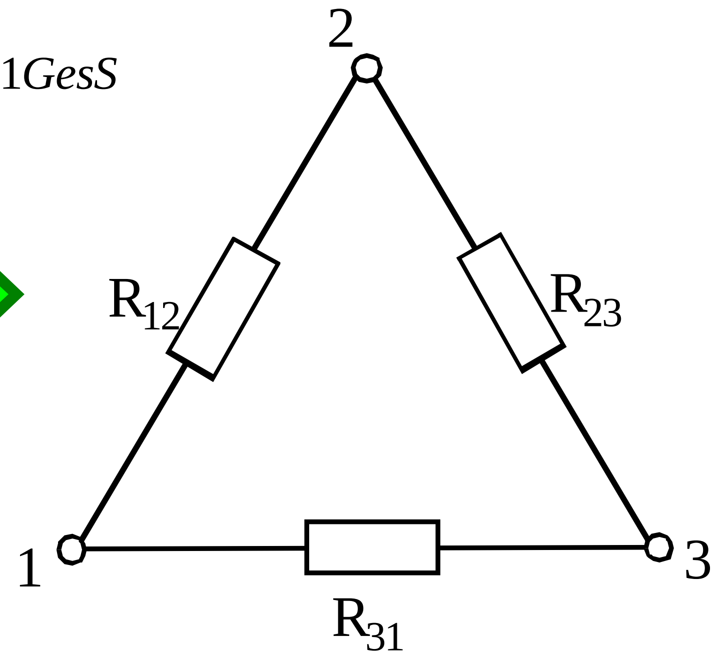
$$R_{23GesD} = \frac{R_{23}\left(R_{12} + R_{31}\right)}{R_{12} + R_{23} + R_{31}}$$

$$R_{23GesD} = \frac{R_{12}R_{23} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{23GesS} = R_{20} + R_{30}$$

$$R_{31GesD} = R_{31GesS}$$







$$R_{31GesS} = R_{10} + R_{30}$$

$$R_{31GesD} = \frac{R_{31}\left(R_{12} + R_{23}\right)}{R_{12} + R_{23} + R_{31}}$$

$$R_{31GesD} = \frac{R_{12}R_{31} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{31GesS} = R_{10} + R_{30}$$

$$R_{12GesD} = \frac{R_{12}R_{23} + R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{12GesS} = R_{10} + R_{20}$$

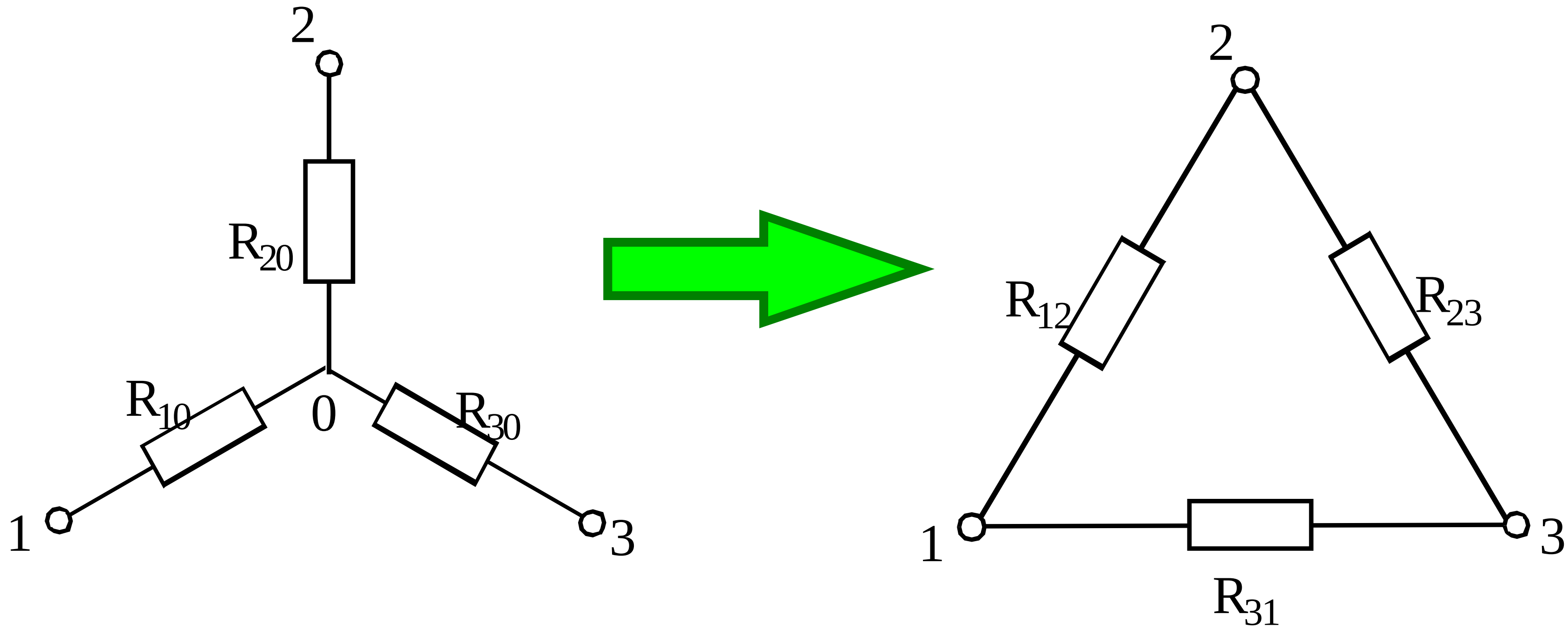
$$R_{31GesD} = \frac{R_{12}R_{31} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{31GesS} = R_{10} + R_{30}$$

$$R_{23GesD} = \frac{R_{12}R_{23} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} = R_{23GesS} = R_{20} + R_{30}$$

$$R_{10} = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_{20} = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_{30} = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$



$$G_{12} = \frac{G_{10}G_{20}}{G_{10} + G_{20} + G_{30}}$$

$$G_{23} = \frac{G_{20}G_{30}}{G_{10} + G_{20} + G_{30}}$$

$$G_{31} = \frac{G_{30}G_{10}}{G_{10} + G_{20} + G_{30}}$$