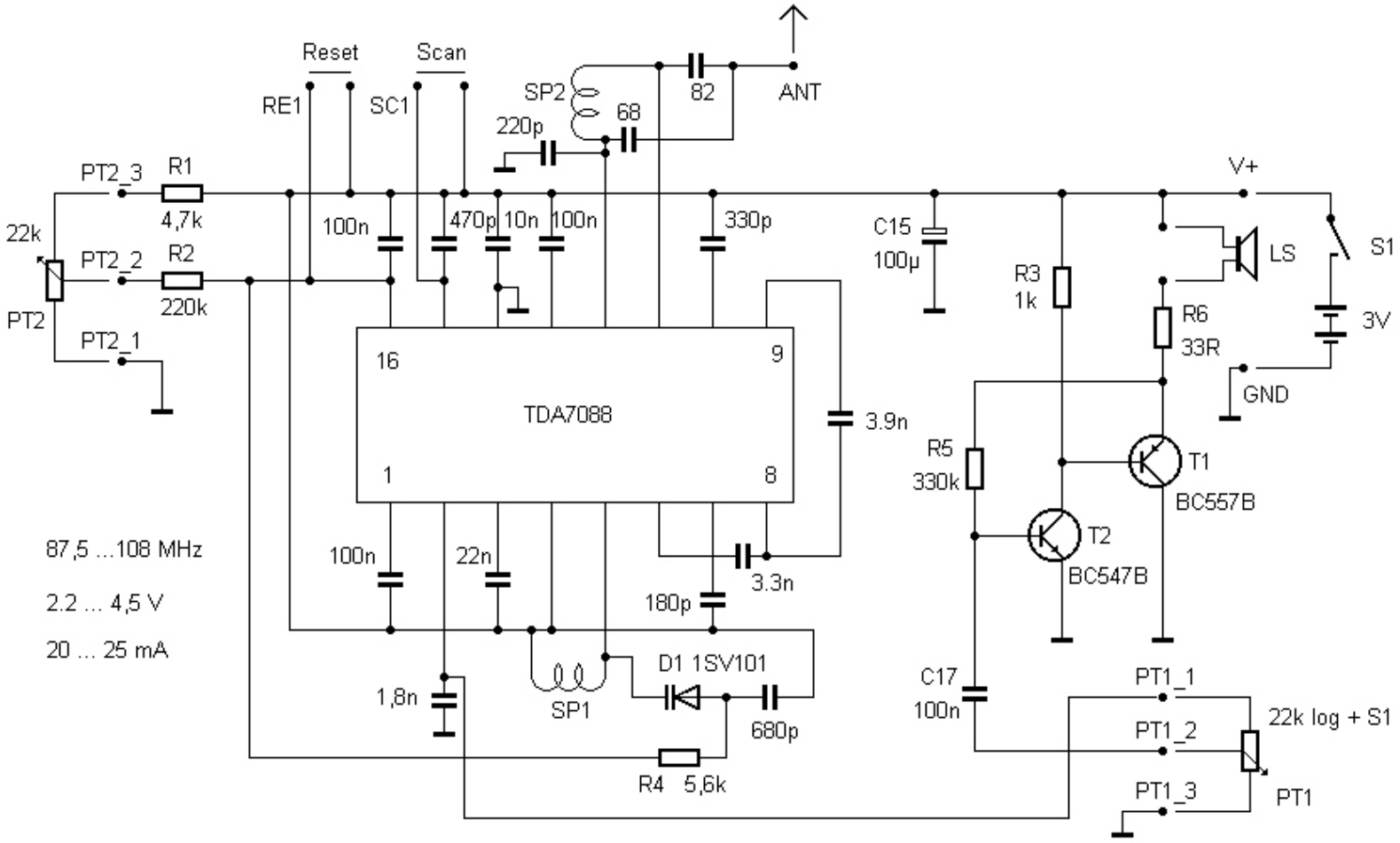
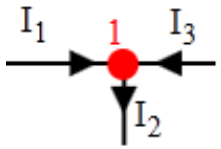
2.6.3 Die Kirchhoffschen Sätze in der Netzwerkberechnung

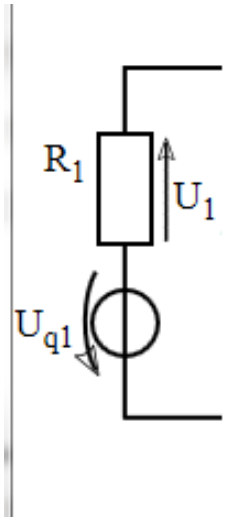


Knoten: Punkt im Netzwerk, in dem eine Stromverzweigung auftritt

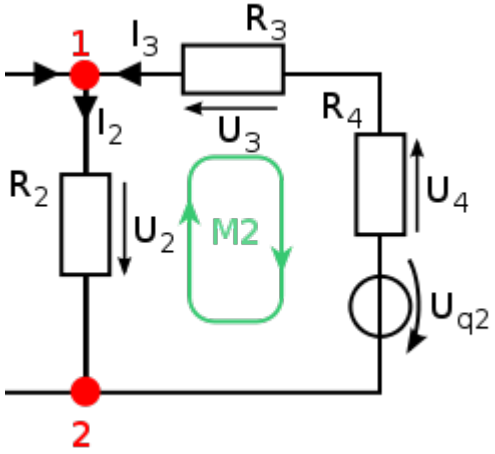
(für Simulationen gilt häufig eine andere Definition, hier wird der Knoten als Verbindung zweier Bauelemente dargestellt!, siehe Pspice, GST)

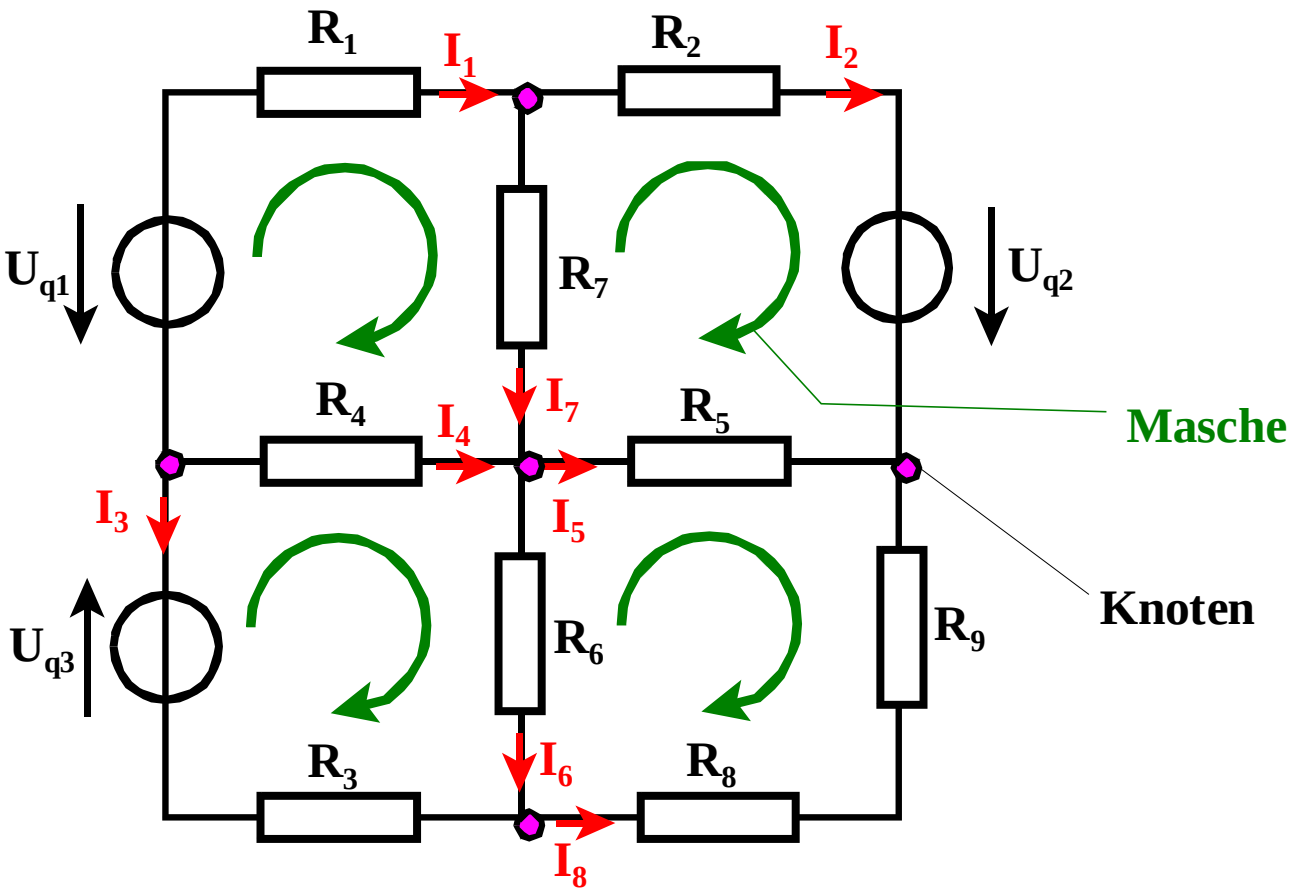


Zweig: Verbindung zweier Knoten durch Zweipolelemente

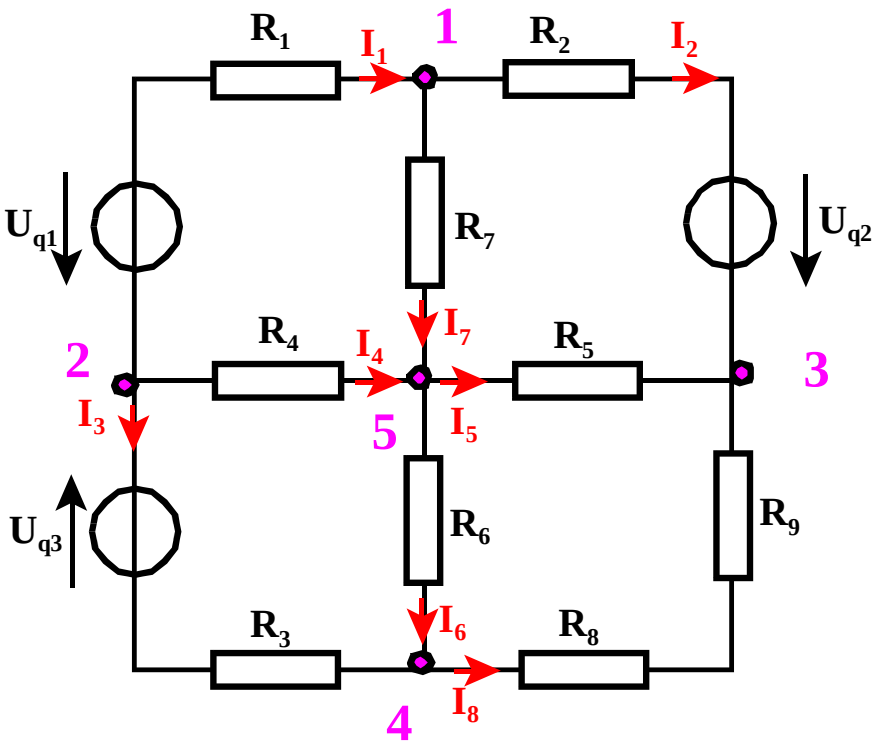


Masche: über Zweige geschlossener Umlauf




R1
I1
R2
I2
Uq1
R7
Uq2
I7
R4
I4
R5
Masche
I3
I5
Knoten
Uq3
R6
R9
R3
I6
R8
I8

Rechenhilfsmittel Knotensatz:



Anzahl der linear unabhängigen Knotengleichungen:

$$\alpha = k-1$$

K1: $I_1 - I_2 - I_7 = 0$

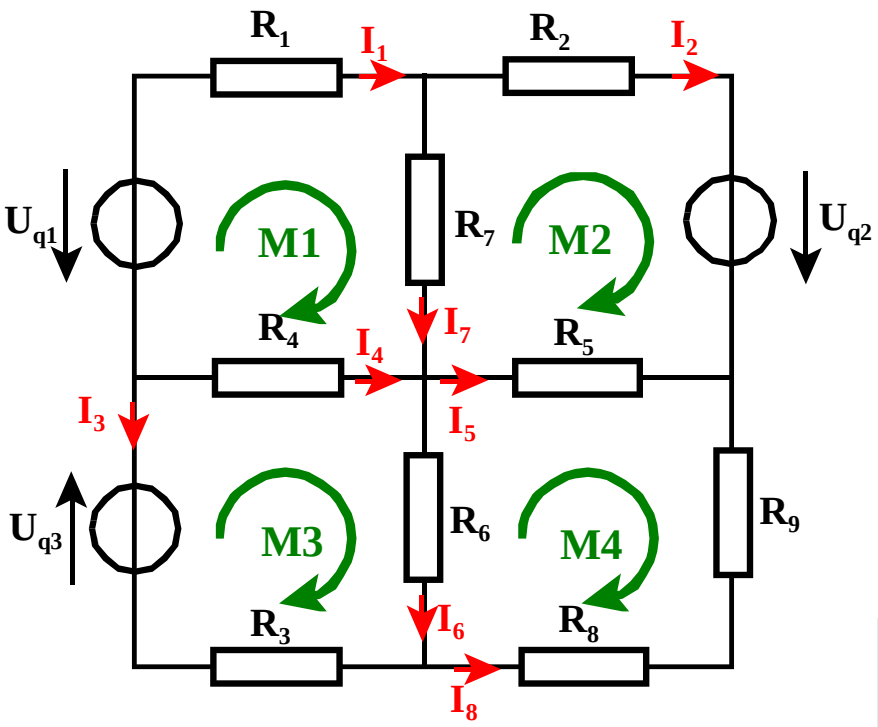
K2: $-I_1 - I_3 - I_4 = 0$

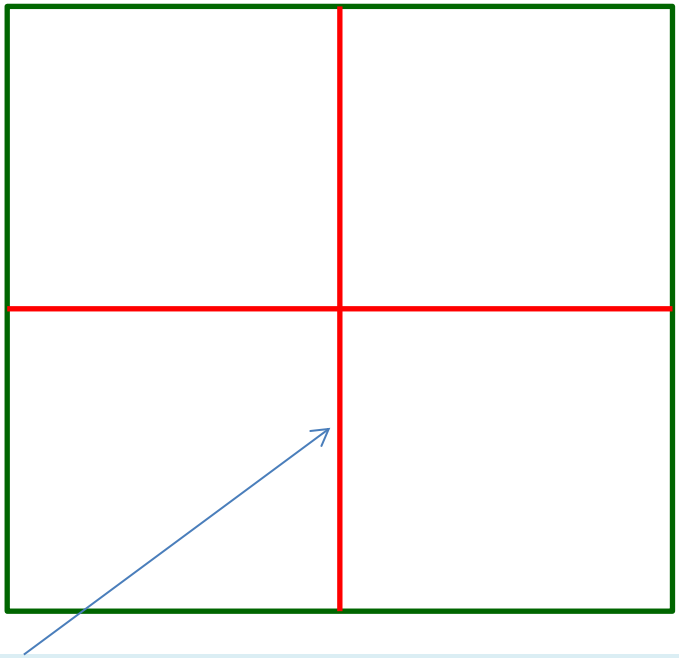
K3: $I_2 + I_5 + I_8 = 0$

K4: $I_3 + I_6 - I_8 = 0$

K5: $I_4 + I_7 - I_5 - I_6 = 0$

Rechenhilfsmittel Maschensatz:





vollständiger Baum (berührt alle Knoten, ohne eine Masche zu bilden)

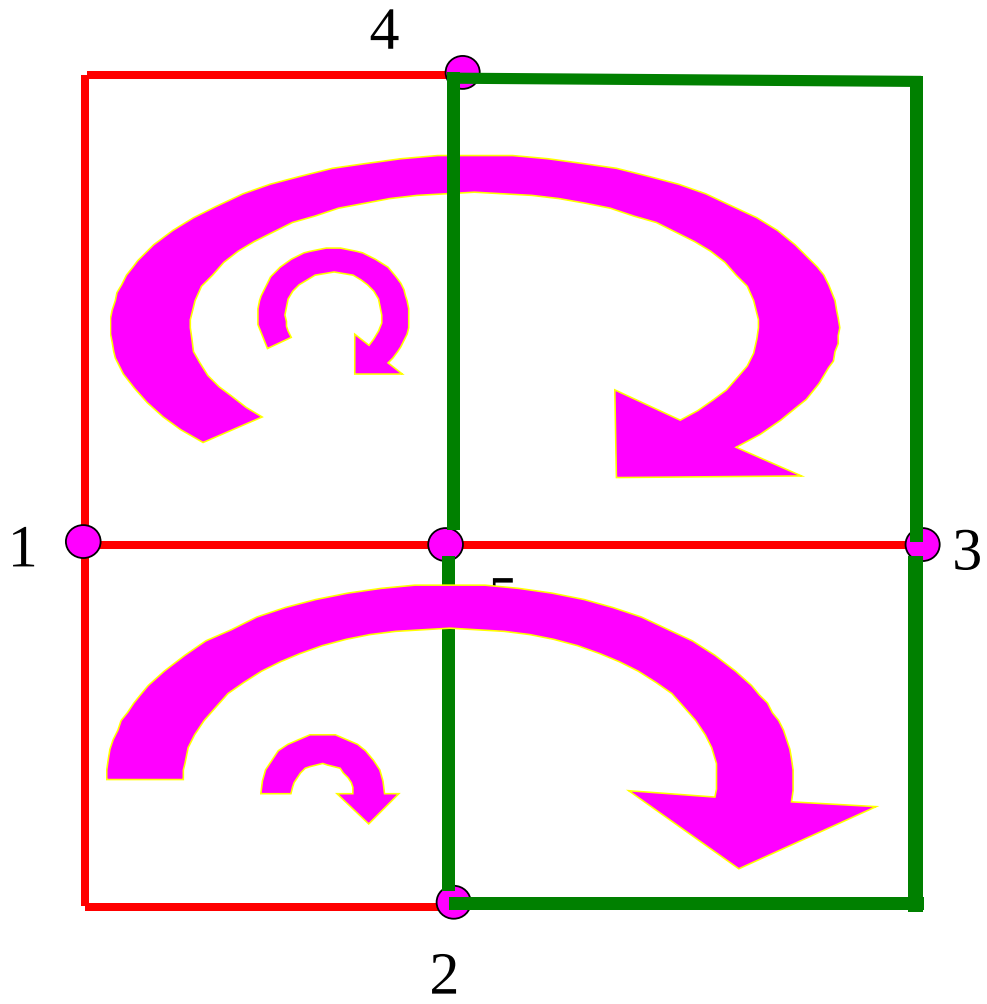
M1: $-U_{q1} + R_1 I_1 + R_7 I_7 - R_4 I_4 = 0$

M2: $U_{q2} - R_5 I_5 - R_7 I_7 + R_2 I_2 = 0$

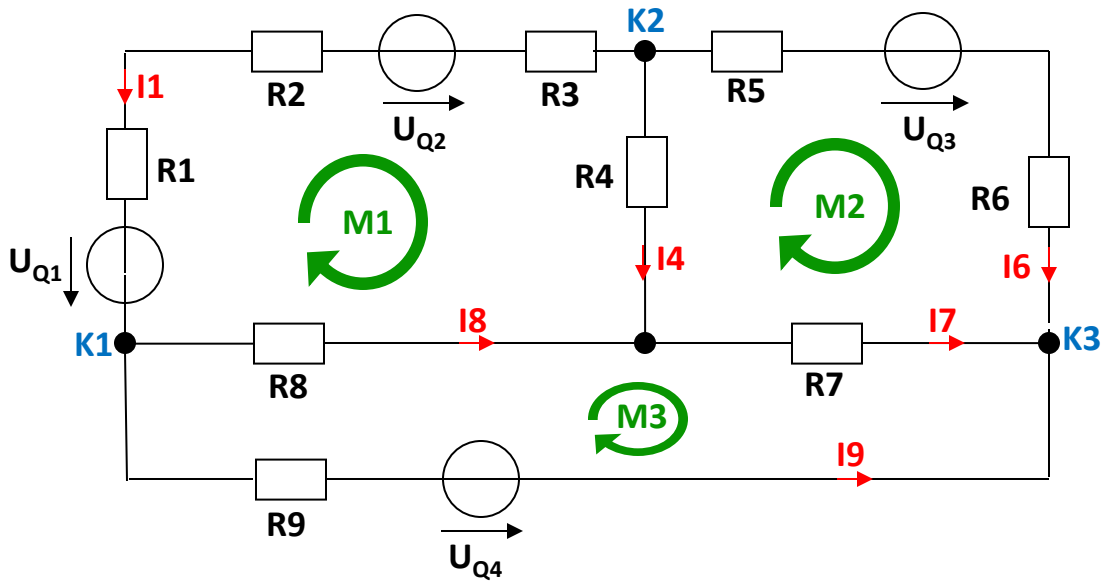
M3: $U_{q3} + R_4 I_4 + R_6 I_6 - R_3 I_3 = 0$

M4: $R_5 I_5 - \left(R_8 + R_9\right) I_8 - R_6 I_6 = 0$

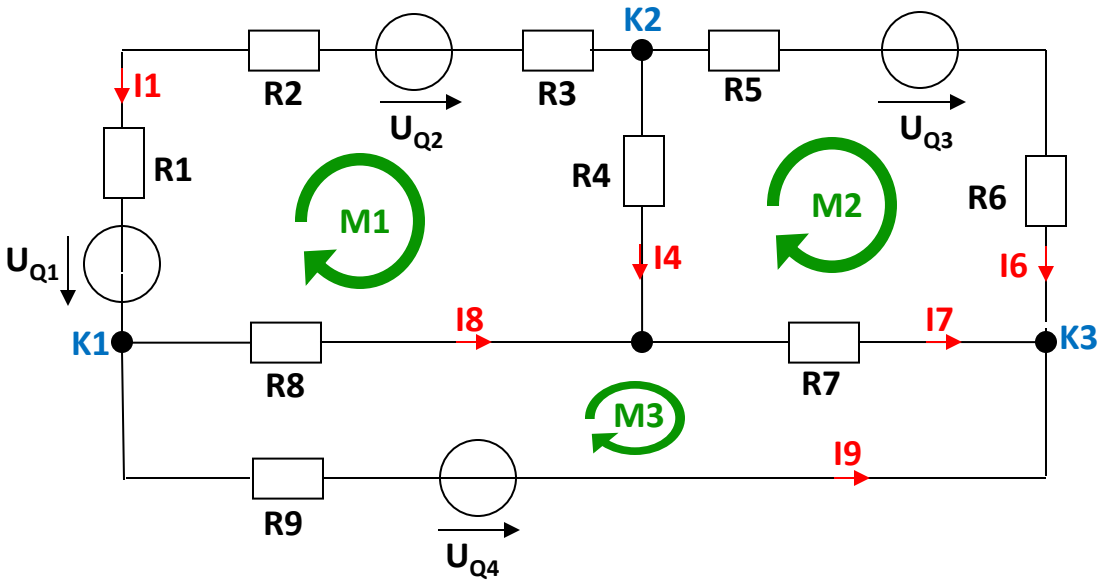
Alternative Struktur:



Beispiel Kirchhoff´sche Sätze



R1 = 100 Ω	R4 = 200 Ω	R7 = 300 Ω	U_{Q1} = 20 V
R2 = 250 Ω	R5 = 400 Ω	R8 = 500 Ω	U_{Q2} = 10 V
R3 = 150 Ω	R6 = 350 Ω	R9 = 700 Ω	U_{Q3} = 30 V
U_{Q4} = 15 V			

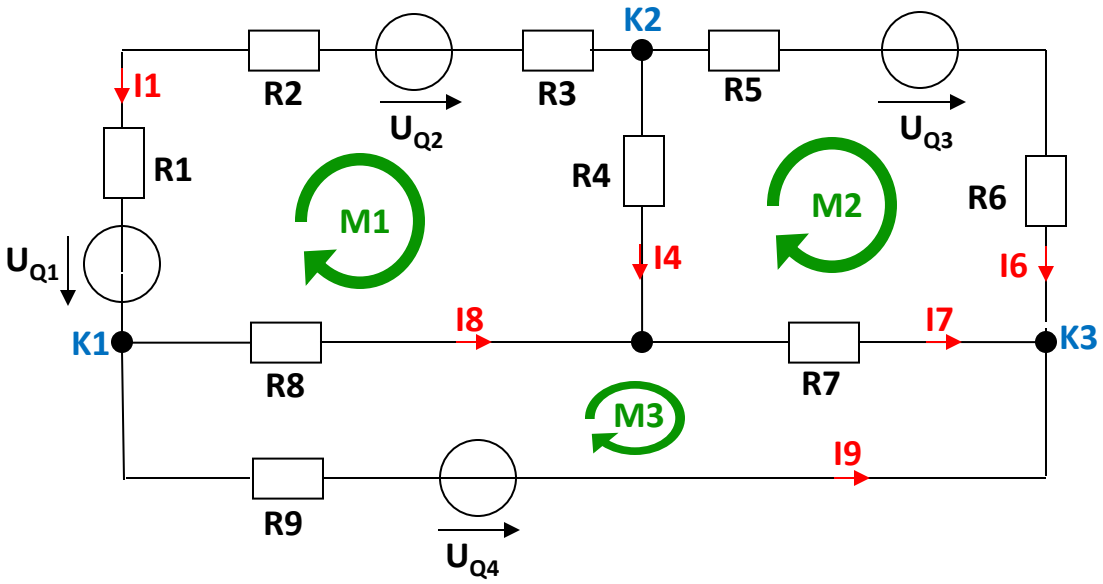


Aufstellen der Knotengleichungen
4 Knoten im Netzwerk -> 3 unabhängige Knotengleichungen

K1: $I1=I8+I9$

K2: $0=I1+I4+I6$

K3: $I6+I7+I9=0$



R1 = 100 Ω
R2 = 250 Ω
R3 = 150 Ω
R4 = 200 Ω
R5 = 400 Ω
R6 = 350 Ω
R7 = 300 Ω
R8 = 500 Ω
R9 = 700 Ω
U_{Q1} = 20 V
U_{Q2} = 10 V
U_{Q3} = 30 V
U_{Q4} = 15 V

Aufstellen der Maschengleichungen

M1: -Uq1+Uq2-I1*(R1+R2+R3)+I4*R4-I8*R8=0

M2: Uq3+I6*(R5+R6)-I7*R7-I4*R4=0

M3: -Uq4-I9*R9+I8*R8+I7*R7 = 0

M1: -500*I1 +200*I4 -500*I8 = 10

M2: 750*I6 -300*I7 -200*I4 = -30

M3: -700*I9 +500*I8 +300*I7 = 15

Maschengleichungen:

M1: -500*I1 +200*I4 -500*I8 = 10

M2: 750*I6 -300*I7 -200*I4 = -30

M3: -700*I9 +500*I8 +300*I7 = 15

Knotengleichungen:

K1: I1=I8+I9

K2: 0=I1+I4+I6

K3: I6+I7+I9=0

Matritzenschreibweise:

$$\begin{matrix} I_1 & I_4 & I_6 & I_7 & I_8 & I_9 \end{matrix}$$

$$\begin{pmatrix} -500 & 200 & 0 & 0 & -500 & 0 \\ 0 & -200 & 750 & -300 & 0 & 0 \\ 0 & 0 & 0 & 300 & 500 & -700 \\ -1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} * \begin{pmatrix} I_1 \\ I_4 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} = \begin{pmatrix} 10 \\ -30 \\ 15 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Maschengleichungen:

M1: -500*I1 +200*I4 -500*I8 = 10

M2: 750*I6 -300*I7 -200*I4 = -30

M3: -700*I9 +500*I8 +300*I7 = 15

Knotengleichungen:

K1: I1=I8+I9

K2: 0=I1+I4+I6

K3: I6+I7+I9=0

Lösung mit Gauß´schem Algorithmus:

Aufstellen der Determinante:

	I1	I4	I6	I7	I8	I9	Uq
M1	-500	200	0	0	-500	0	10
K2	1	1	1	0	0	0	0
K1	-1	0	0	0	1	1	0
M2	0	-200	750	-300	0	0	-30
K3	0	0	1	1	0	1	0
M3	0	0	0	300	500	-700	15

	I1	I4	I6	I7	I8	I9	Uq
	-500	200	0	0	-500	0	10
*500	1	1	1	0	0	0	0
*-500	-1	0	0	0	1	1	0
	0	-200	750	-300	0	0	-30
	0	0	1	1	0	1	0
	0	0	0	300	500	-700	15

I1	I4	I6	I7	I8	I9	Uq
-500	200	0	0	-500	0	10
0	700	500	0	-500	0	10
0	200	0	0	-1000	-500	10
0	-200	750	-300	0	0	-30
0	0	1	1	0	1	0
0	0	0	300	500	-700	15

*-3,5

*3,5

I1	I4	I6	I7	I8	I9	Uq
-500	200	0	0	-500	0	10
0	700	500	0	-500	0	10
0	0	500	0	3000	1750	-25
0	0	3125	-1050	-500	0	-95
0	0	1	1	0	1	0
0	0	0	300	500	-700	15

/-6,25

*-500

I1	I4	I6	I7	I8	I9	Uq	
-500	200	0	0	-500	0	10	
0	700	500	0	-500	0	10	
0	0	500	0	3000	1750	-25	
0	0	3125	-1050	-500	0	-95	/-6,25
0	0	1	1	0	1	0	*-500
0	0	0	300	500	-700	15	

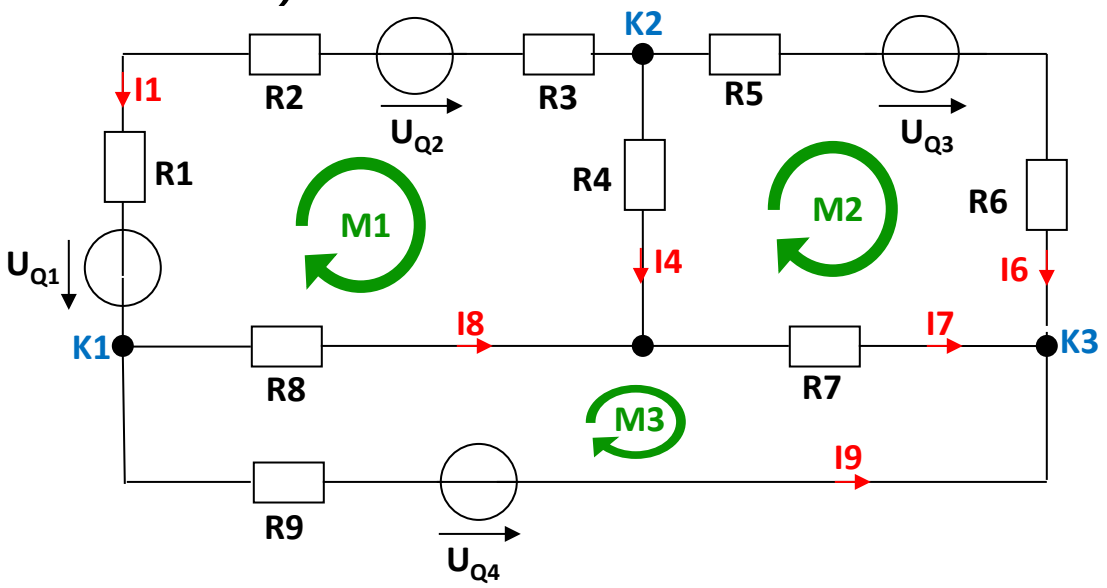
I1	I4	I6	I7	I8	I9	Uq	
-500	200	0	0	-500	0	10	
0	700	500	0	-500	0	10	
0	0	500	0	3000	1750	-25	
0	0	0	168	3080	1750	-9,8	
0	0	0	-500	3000	1250	-25	*0,336
0	0	0	300	500	-700	15	*-0,56

I1	I4	I6	I7	I8	I9	Uq	
-500	200	0	0	-500	0	10	
0	700	500	0	-500	0	10	
0	0	500	0	3000	1750	-25	
0	0	0	168	3080	1750	-9,8	
0	0	0	0	4088	2170	-18,2	
0	0	0	0	2800	2142	-18,2	*-1,46

I1	I4	I6	I7	I8	I9	Uq
-500	200	0	0	-500	0	10
0	700	500	0	-500	0	10
0	0	500	0	3000	1750	-25
0	0	0	168	3080	1750	-9,8
0	0	0	0	4088	2170	-18,2
0	0	0	0	0	-957,32	8,372

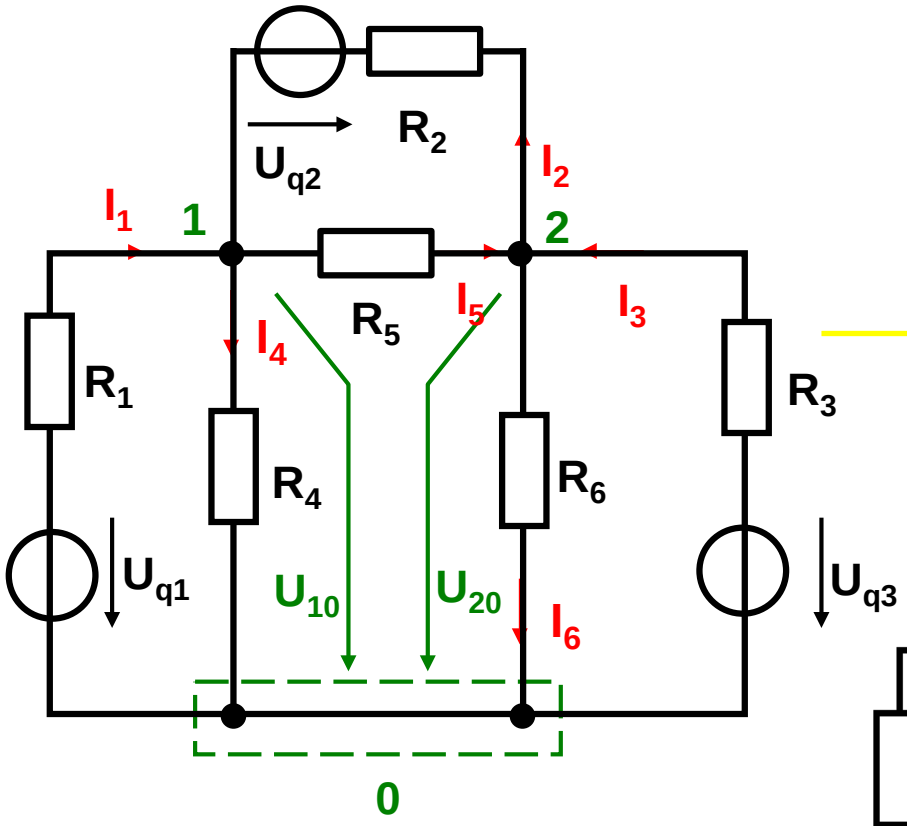
Lösungen (alle Angaben in mA)

I9	-8,745
I8	0,190
I7	29,278
I6	-20,532
I4	29,087
I1	-8,555



2.6.4 Die Methode der Knotenspannungen

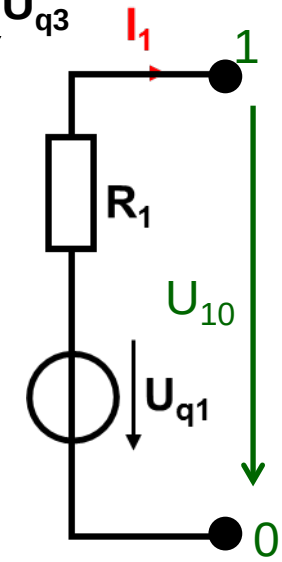
(bisher 6 Unbekannte → Ziel: 2 Unbekannte)



Über R_4 und R_6 fällt die Knotenspannung U_{10} bzw. U_{20} ab, somit ergeben sich die Ströme mittels ohmschen Gesetz zu:

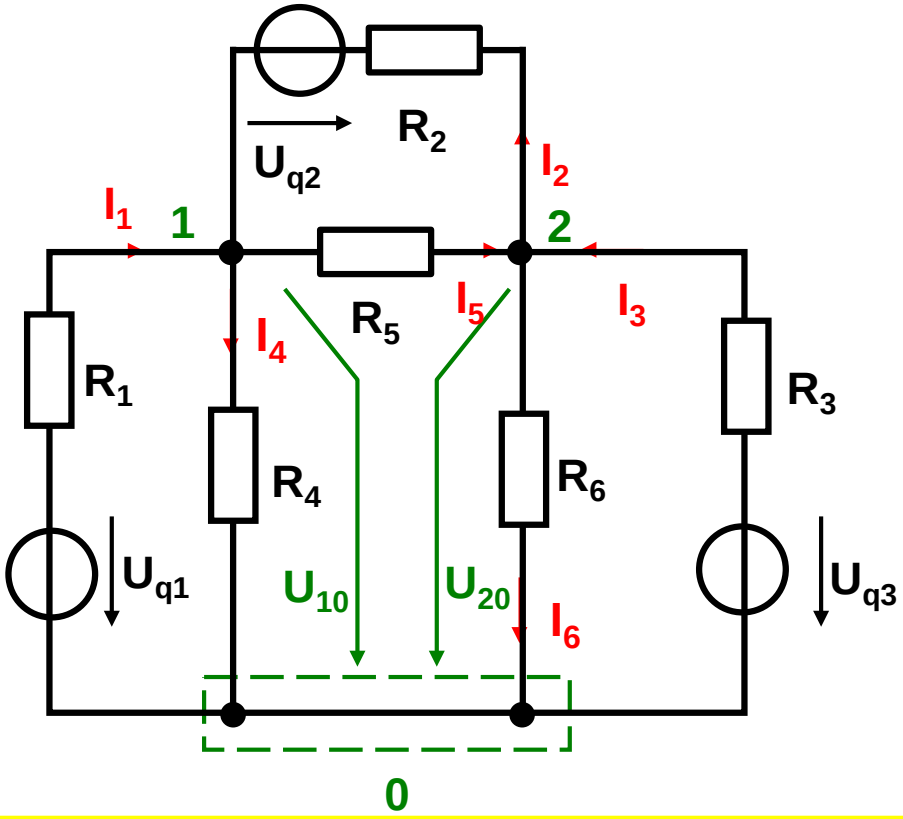
$$I_4 = \frac{U_{10}}{R_4} \qquad I_6 = \frac{U_{20}}{R_6}$$

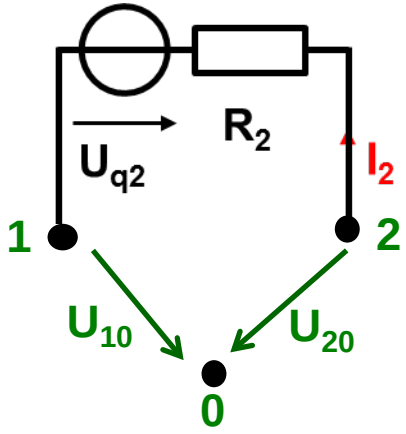
Bei allen anderen Strömen müssen die Quellen bzw. mehrere Knotenspannungen berücksichtigt werden, hier kommt der Maschensatz zur Anwendung:



$$U_{10} - U_{q1} + I_1 R_1 = 0$$

$$I_1 = \frac{U_{q1} - U_{10}}{R_1}$$





$$-U_{10} + U_{q2} - I_2 R_2 + U_{20} = 0$$

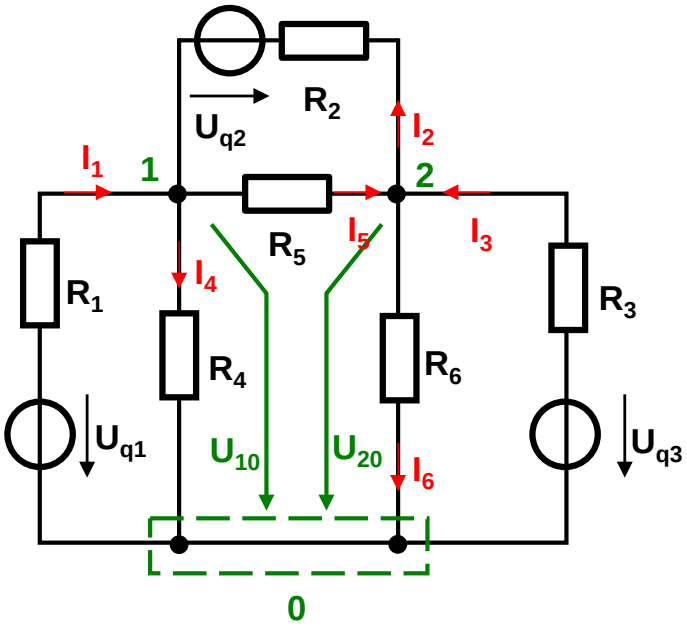
$$I_2 = \frac{U_{20} - U_{10} + U_{q2}}{R_2}$$

$$-U_{20} + U_{q3} - I_3 R_3 = 0$$

$$I_3 = \frac{U_{q3} - U_{20}}{R_3}$$

$$-U_{10} + I_5 R_5 + U_{20} = 0$$

$$I_5 = \frac{U_{10} - U_{20}}{R_5}$$



Knoten 1 : $I_1 + I_2 - I_4 - I_5 = 0$

$$\frac{U_{q1} - U_{10}}{R_1} + \frac{U_{q2} + U_{20} - U_{10}}{R_2} - \frac{U_{10}}{R_4} - \frac{U_{10} - U_{20}}{R_5} = 0$$

$$\frac{U_{q1}}{R_1} + \frac{U_{q2}}{R_2} = U_{10}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - U_{20}\left(\frac{1}{R_2} + \frac{1}{R_5}\right)$$

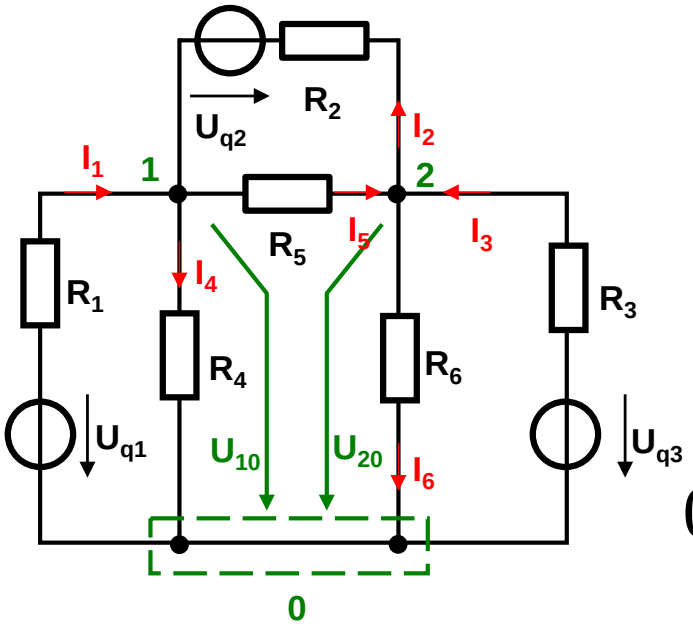
Knoten 2: $-I_2 + I_3 + I_5 - I_6 = 0$

$$-\frac{U_{q2}}{R_2} + \frac{U_{q3}}{R_3} = U_{20}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_6}\right) - U_{10}\left(\frac{1}{R_2} + \frac{1}{R_5}\right)$$

$$I_1 = \frac{U_{q1} - U_{10}}{R_1} \qquad I_2 = \frac{U_{20} - U_{10} + U_{q2}}{R_2}$$

$$I_3 = \frac{U_{q3} - U_{20}}{R_3} \qquad I_4 = \frac{U_{10}}{R_4}$$

$$I_5 = \frac{U_{10} - U_{20}}{R_5} \qquad I_6 = \frac{U_{20}}{R_6}$$



$$\frac{U_{q1}}{R_1}+\frac{U_{q2}}{R_2}=U_{10}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)-U_{20}\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$

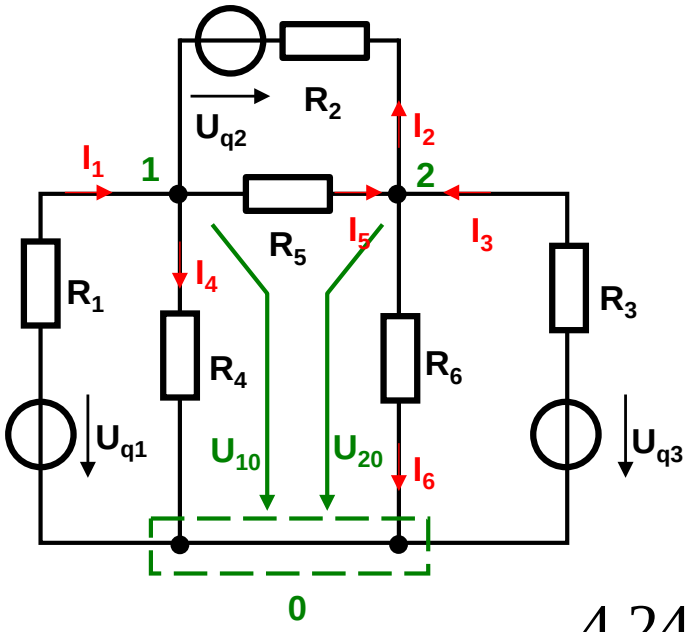
$$-\frac{U_{q2}}{R_2}+\frac{U_{q3}}{R_3}=U_{20}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_6}\right)-U_{10}\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$

$$0{,}2=0{,}0208*U_{10}-0{,}0125*U_{20}$$

$$0{,}067=0{,}0212*U_{20}-0{,}0125*U_{10}$$

$$\begin{pmatrix}0{,}0208 & -0{,}0125\\ -0{,}0125 & 0{,}0212\end{pmatrix}*\begin{pmatrix}U_{10}\\ U_{20}\end{pmatrix}=\begin{pmatrix}0{,}2\\ 0{,}067\end{pmatrix}$$

R1 = 200 Ω
R2 = 100 Ω
R3 = 150 Ω
R4 = 300 Ω
R5 = 400 Ω
R6 = 500 Ω
U_{Q1} = 20 V
U_{Q2} = 10 V
U_{Q3} = 25 V



$$\begin{pmatrix} 0{,}0208 & -0{,}0125 \\ -0{,}0125 & 0{,}0212 \end{pmatrix} * \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} 0{,}2 \\ 0{,}067 \end{pmatrix}$$

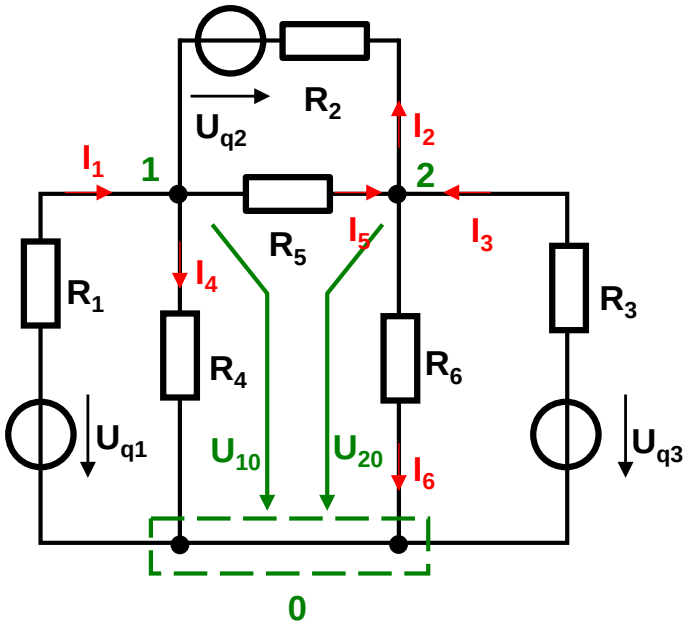
$$U_{10} = \frac{\begin{vmatrix} 0{,}2 & -0{,}0125 \\ 0{,}067 & 0{,}0212 \end{vmatrix}}{\begin{vmatrix} 0{,}0208 & -0{,}0125 \\ -0{,}0125 & 0{,}0212 \end{vmatrix}}$$

$$U_{10} = \frac{4{,}24 * 10^{-3} - -0{,}8375 * 10^{-3}}{0{,}441 * 10^{-3} - 0{,}156 * 10^{-3}} = \frac{5{,}0775 * 10^{-3}}{0{,}285 * 10^{-3}} = 17{,}82V$$

R1 = 200 Ω
R2 = 100 Ω
R3 = 150 Ω
R4 = 300 Ω
R5 = 400 Ω
R6 = 500 Ω
U_{Q1} = 20 V
U_{Q2} = 10 V
U_{Q3} = 25 V

$$U_{20} = \frac{\begin{vmatrix} 0{,}0208 & 0{,}2 \\ -0{,}0125 & 0{,}067 \end{vmatrix}}{\begin{vmatrix} 0{,}0208 & -0{,}0125 \\ -0{,}0125 & 0{,}0212 \end{vmatrix}}$$

$$U_{20} = \frac{1{,}3936 * 10^{-3} - -2{,}5 * 10^{-3}}{0{,}441 * 10^{-3} - 0{,}156 * 10^{-3}} = \frac{3{,}8936 * 10^{-3}}{0{,}285 * 10^{-3}} = 13{,}66V$$



$$U_{10} = 17{,}82V$$

$$U_{20} = 13{,}66V$$

R1 = 200 Ω
R2 = 100 Ω
R3 = 150 Ω
R4 = 300 Ω
R5 = 400 Ω
R6 = 500 Ω
U_{Q1} = 20 V
U_{Q2} = 10 V
U_{Q3} = 25 V

$$I_4 = \frac{U_{10}}{R_4} = 59{,}3 \text{ mA}$$

$$I_6 = \frac{U_{20}}{R_6} = 27{,}3 \text{ mA}$$

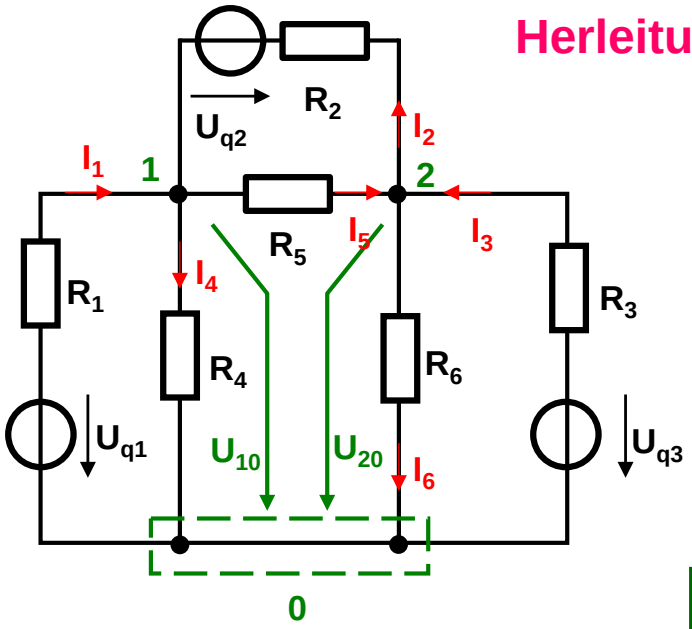
$$I_5 = \frac{U_{10} - U_{20}}{R_5} = 10{,}3 \text{ mA}$$

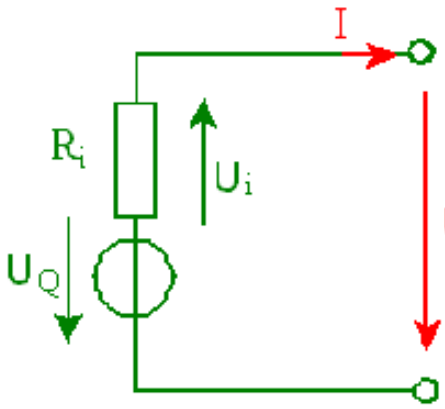
$$I_2 = \frac{U_{20} - U_{10} + U_{q2}}{R_2} = 58{,}8 \text{ mA}$$

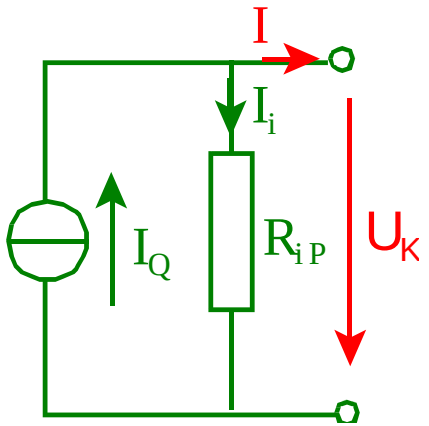
$$I_3 = \frac{U_{q3} - U_{20}}{R_3} = 75{,}6 \text{ mA}$$

$$I_1 = \frac{U_{q1} - U_{10}}{R_1} = 11{,}1 \text{ mA}$$

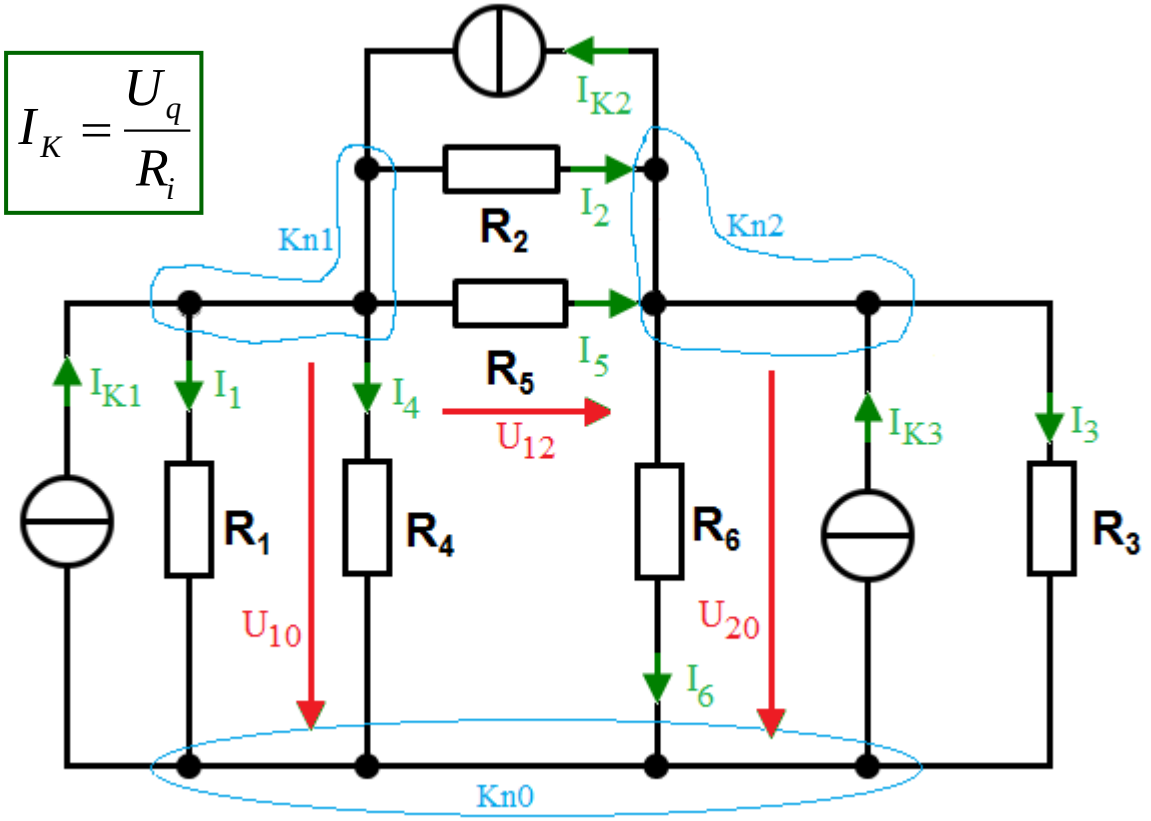
Herleitung einer allgemeinen Beziehung über Stromquellen:

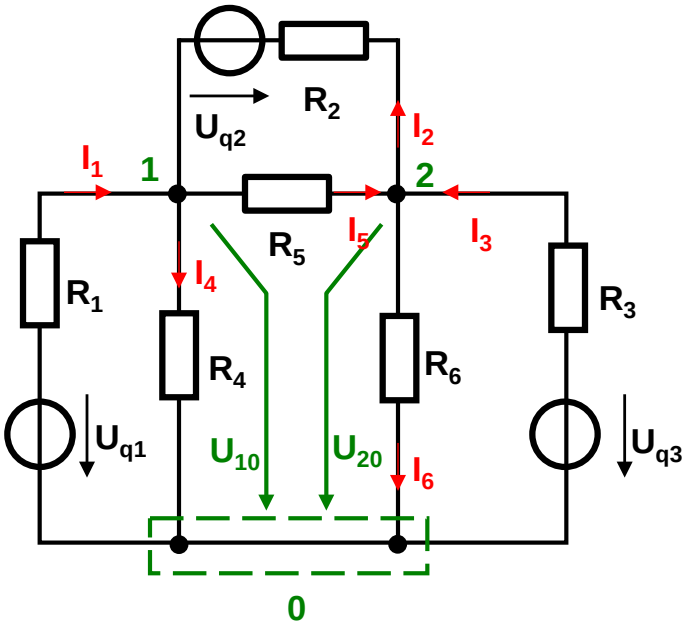




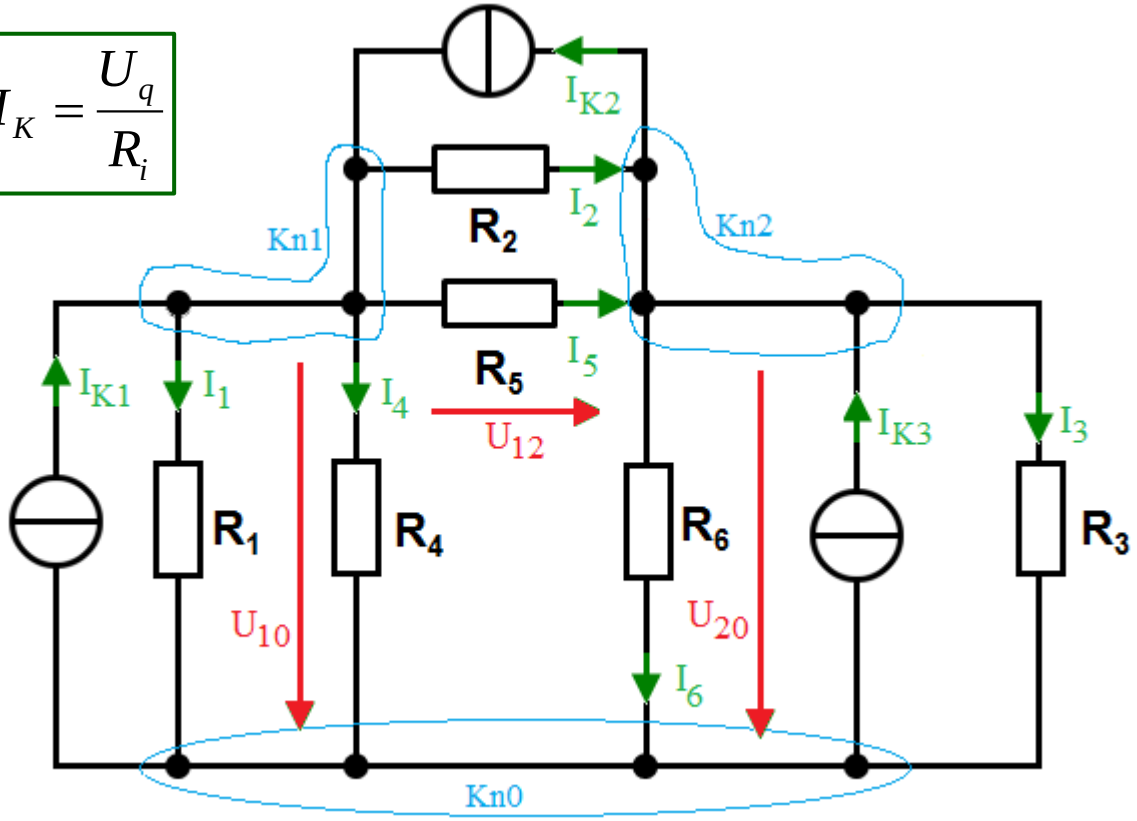


$$I_K = \frac{U_q}{R_i}$$





$$I_K = \frac{U_q}{R_i}$$



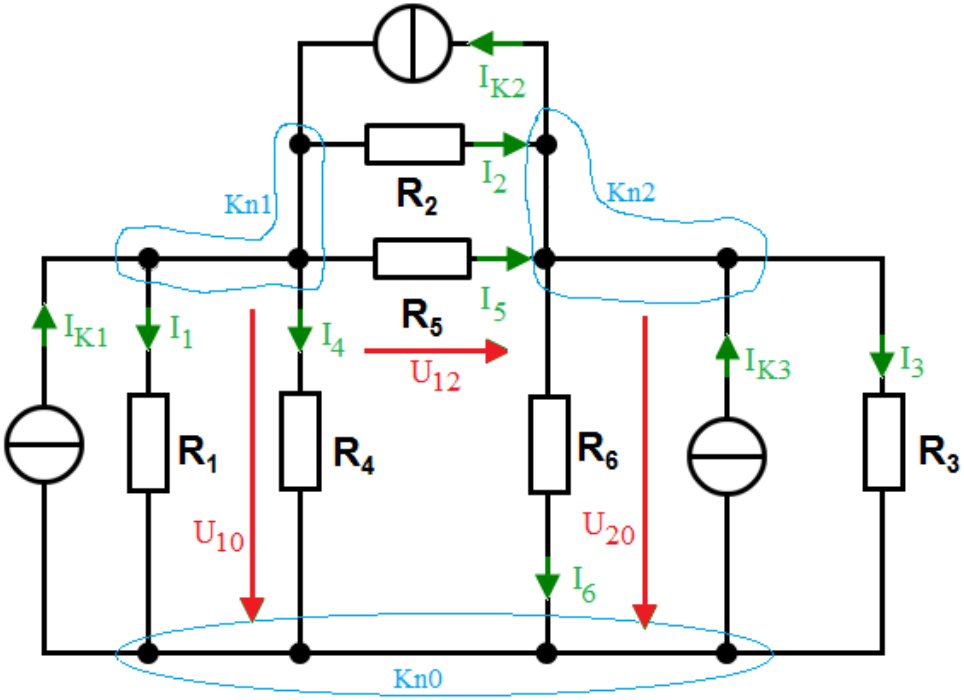
Für Knoten 1 gilt:

$$I_{K1} + I_{K2} = I_1 + I_2 + I_4 + I_5$$

$$\frac{U_{q1}}{R_1} + \frac{U_{q2}}{R_2} = \frac{U_{10}}{R_1} + \frac{U_{10}}{R_4} + \frac{U_{12}}{R_2} + \frac{U_{12}}{R_5}$$

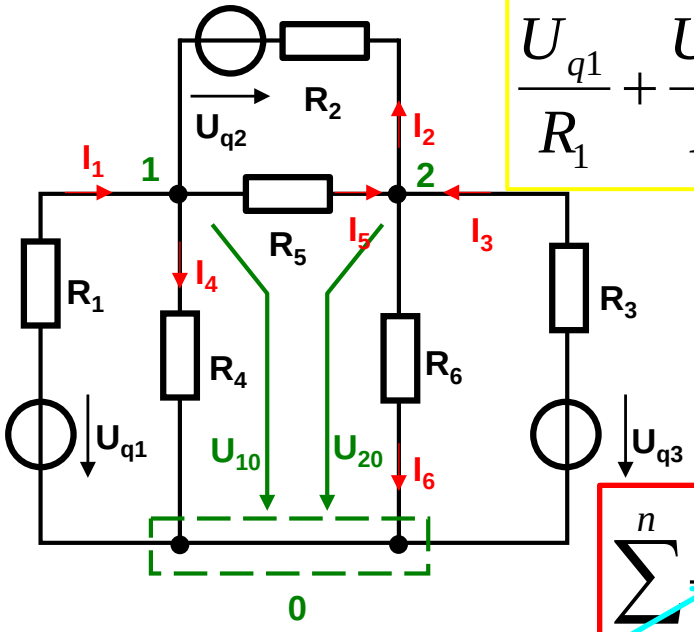
$$U_{12} = U_{10} - U_{20}$$

$$\frac{U_{q1}}{R_1} + \frac{U_{q2}}{R_2} = \frac{U_{10}}{R_1} + \frac{U_{10}}{R_4} + \frac{U_{10} - U_{20}}{R_2} + \frac{U_{10} - U_{20}}{R_5}$$



$$\frac{U_{q1}}{R_1}+\frac{U_{q2}}{R_2}=\frac{U_{10}}{R_1}+\frac{U_{10}}{R_4}+\frac{U_{10}-U_{20}}{R_2}+\frac{U_{10}-U_{20}}{R_5}$$

$$\frac{U_{q1}}{R_1}+\frac{U_{q2}}{R_2}=U_{10}\cdot\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_2}+\frac{1}{R_5}\right)-U_{20}\cdot\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$



$$\frac{U_{q1}}{R_1}+\frac{U_{q2}}{R_2}=U_{10}\cdot\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_2}+\frac{1}{R_5}\right)-U_{20}\cdot\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$

Vereinfachtes Verfahren:

1. Bezugsknoten festlegen
2. für jeden weiteren Knoten gilt:

$$\sum_{i=1}^{n}\frac{U_{QZweig}}{R_{Zweig}}=U_{j0}\cdot\sum_{k=1}^{m}\frac{1}{R_{Zweig}}-\sum_{l=1}^{o}U_{benachbart}\cdot\frac{1}{R_{koppel}}$$

vorzeichenbehaftete Kurzschlussströme aller den Knoten berührender Zweige

U_q von Knoten weg: +

U_q auf Knoten zu: -

Leitwerte aller den Knoten berührender Zweige

Knotenspannung des betrachteten Knotens

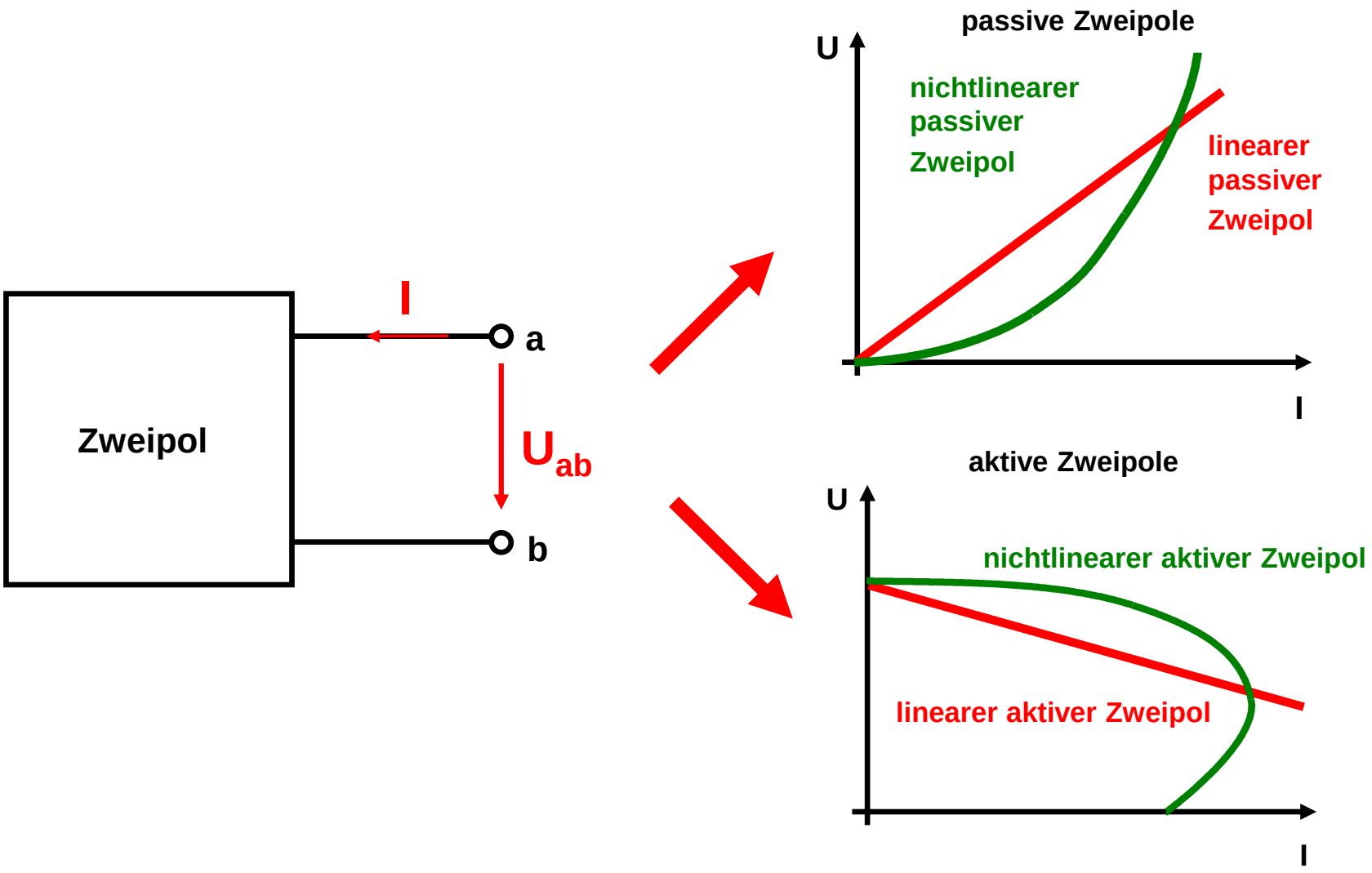
Knotenspannung der/des benachbarten Knoten(s)

Leitwert der/des Koppelzweige(s)

$$\frac{U_{q1}}{R_1}+\frac{U_{q2}}{R_2}=U_{10}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)-U_{20}\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$

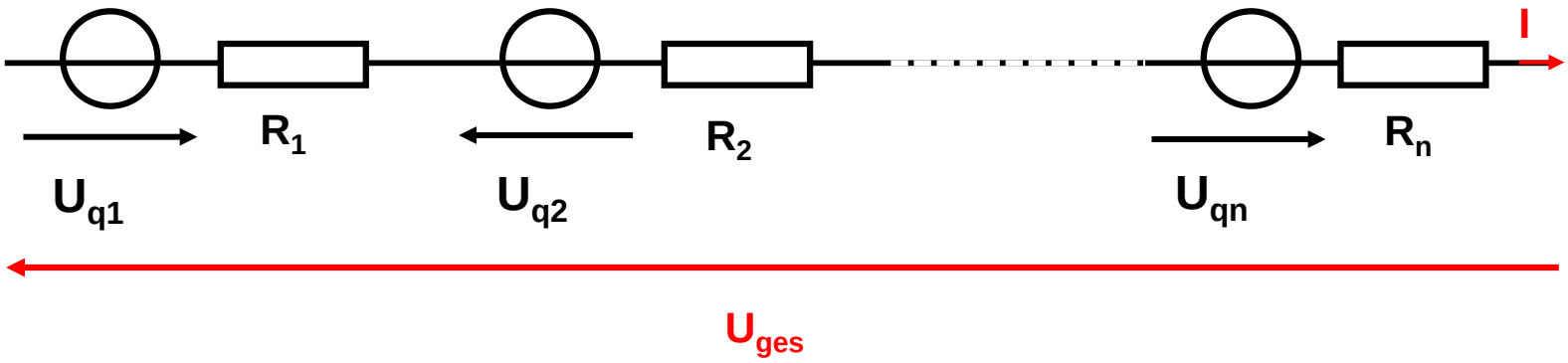
$$-\frac{U_{q2}}{R_2}+\frac{U_{q3}}{R_3}=U_{20}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_6}\right)-U_{10}\left(\frac{1}{R_2}+\frac{1}{R_5}\right)$$

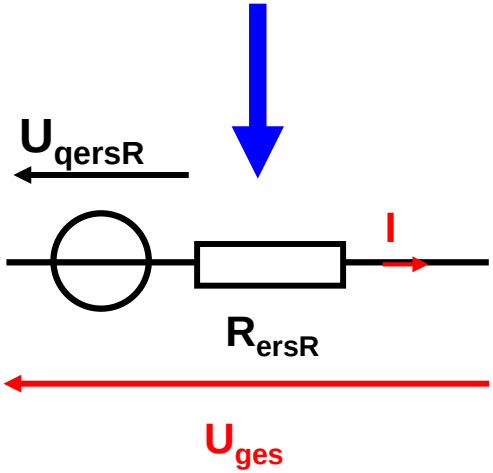
2.6.5 Die Zweipoltheorie



Berechnung linearer aktiver Zweipole

- Reihenschaltung:

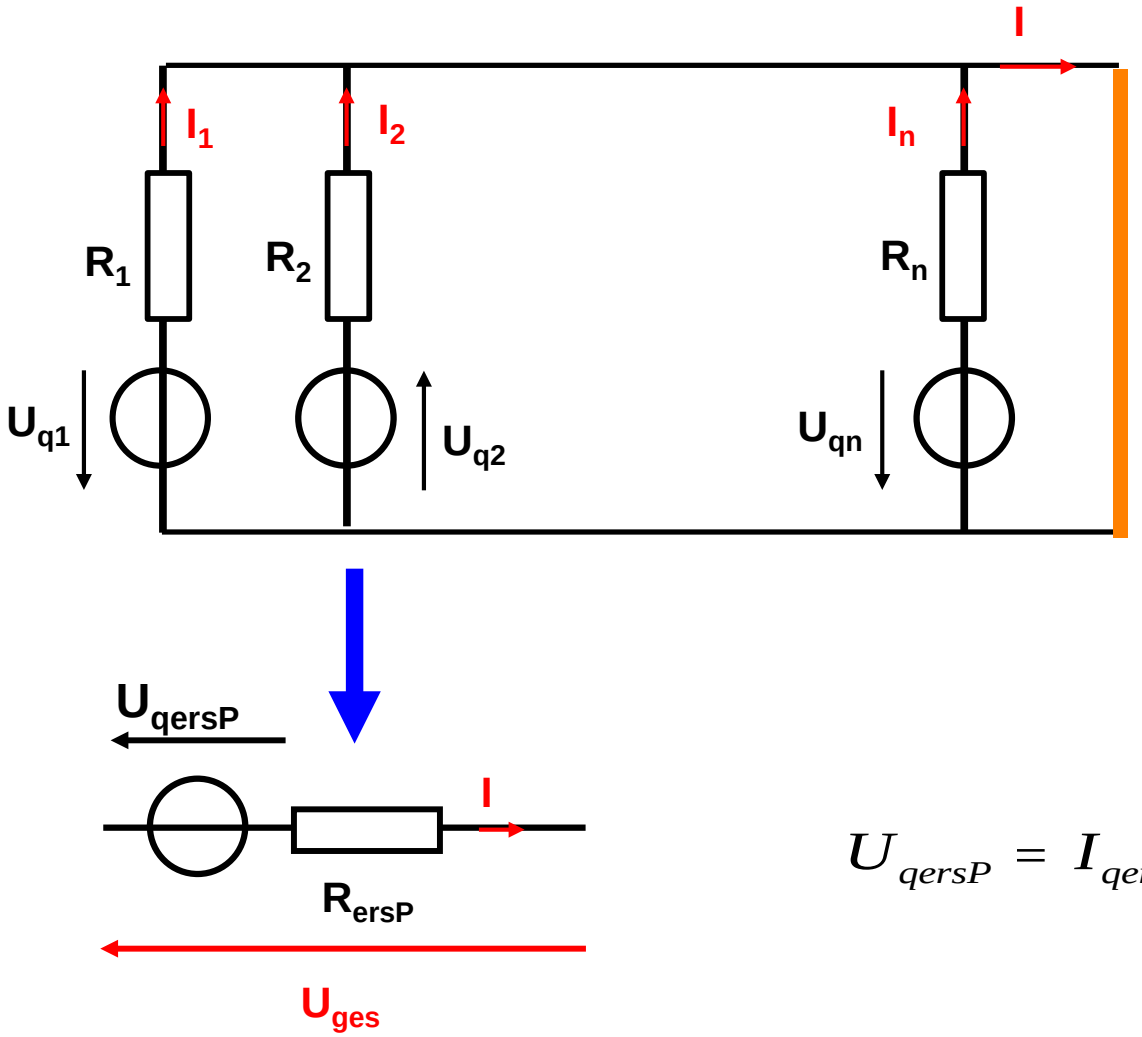




$$R_{ersR} = \sum_{i=1}^{n} R_i$$

$$U_{qersR} = U_{gesL} = -U_{q1} + U_{q2} + \cdots - U_{qn} = \sum_{i=1}^{n} U_{qi\,vorz}$$

- *Parallelschaltung:*



$$\frac{1}{R_{ersP}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

$$\mp U_{qi} + R_i I_{Ki} = 0$$

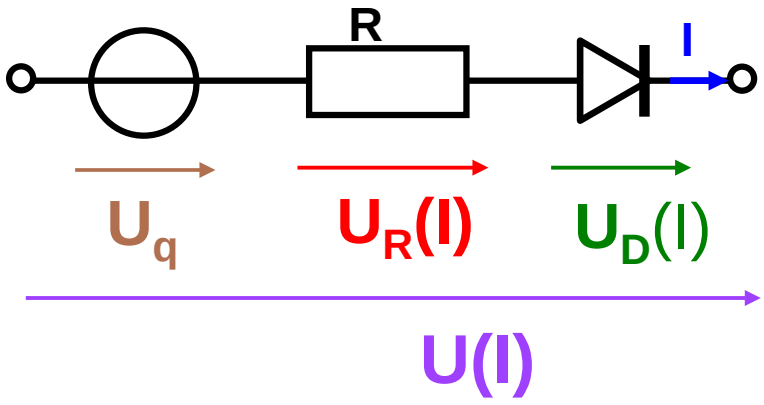
$$I_{qers} = I_K = \sum_{i=1}^{n} I_{Ki\,vorz} = \sum_{i=1}^{n} \frac{U_{qi\,vorz}}{R_i}$$

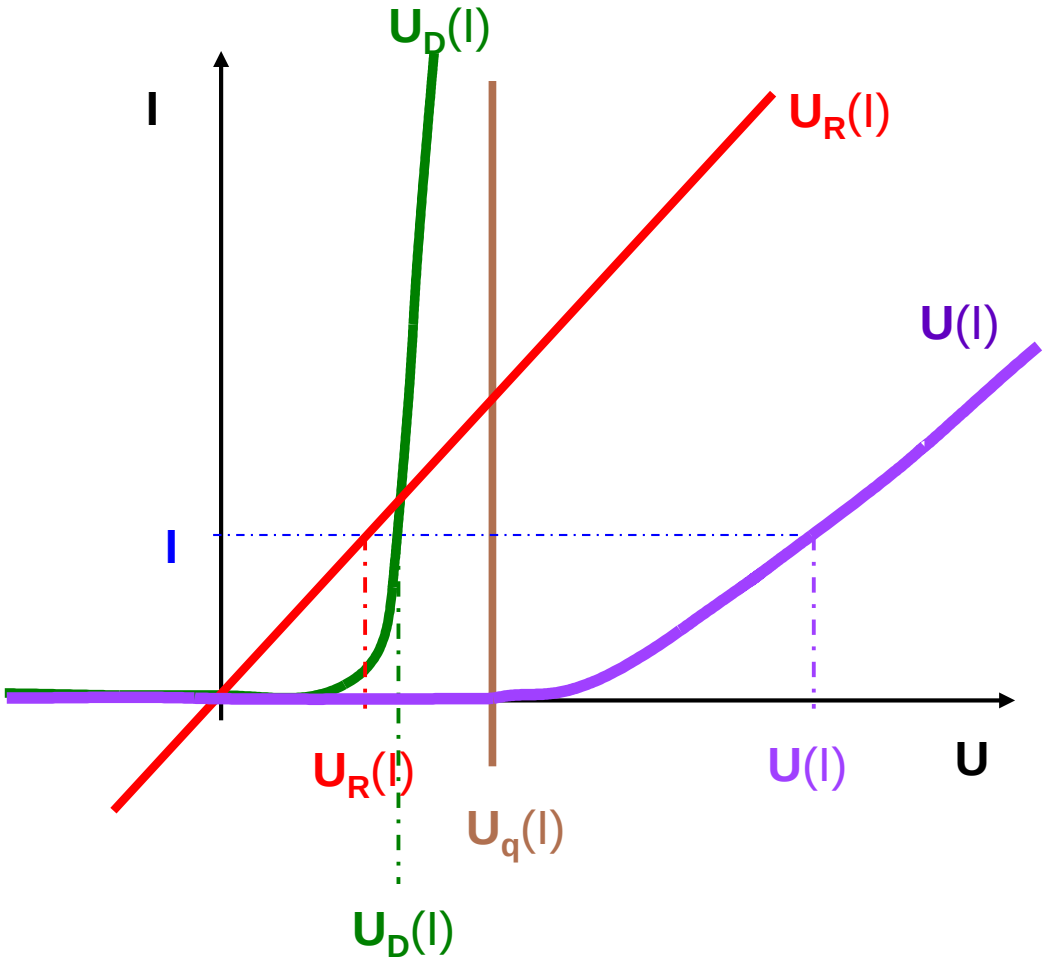
$$I_{qers} = \frac{U_{q1}}{R_1} - \frac{U_{q2}}{R_2} + \cdots + \frac{U_{qn}}{R_n}$$

$$U_{qersP} = I_{qers} R_{ersP} = \frac{\sum_{i=1}^{n} \frac{U_{qi\,vorz}}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

Berechnung nichtlinearer Zweipole

- Reihenschaltung:



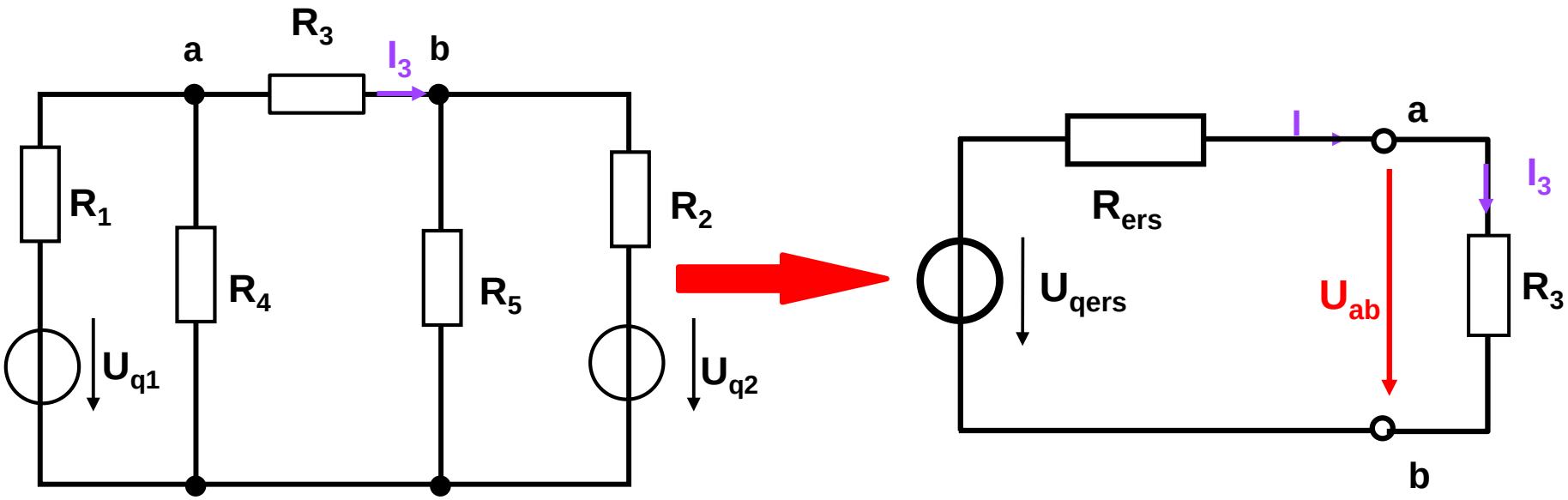


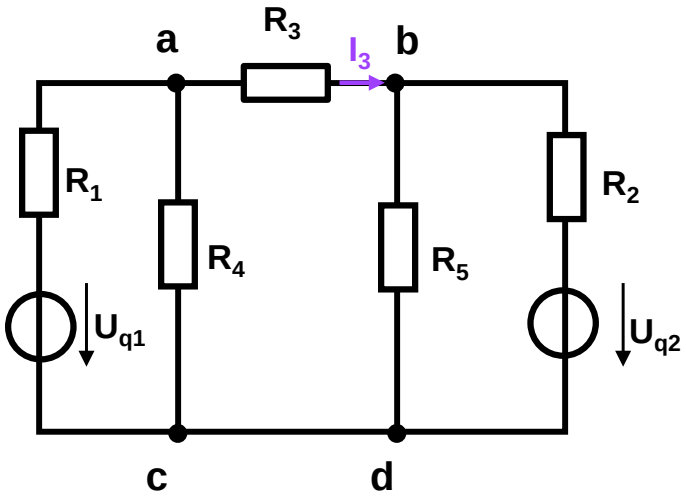
$$I = I_Q = I_R = I_D$$

$$U(I) = U_q(I) + U_R(I) + U_D(I)$$

Netzwerkberechnung mittels Zweipoltheorie

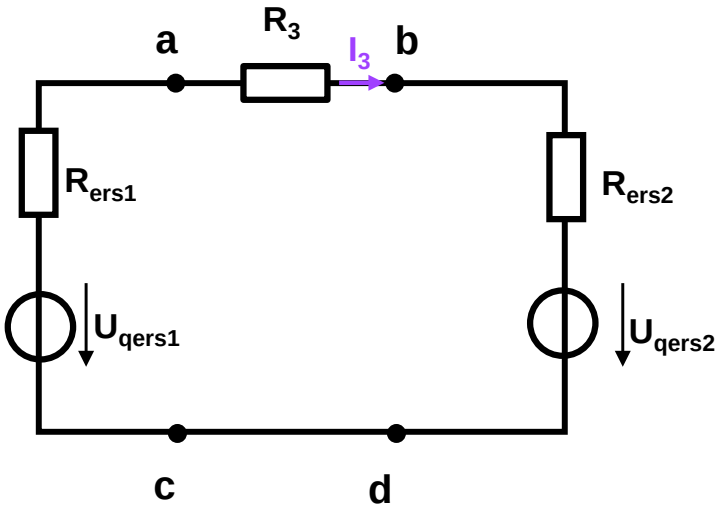
Aufgabe:





Berechnung von R_{ers}

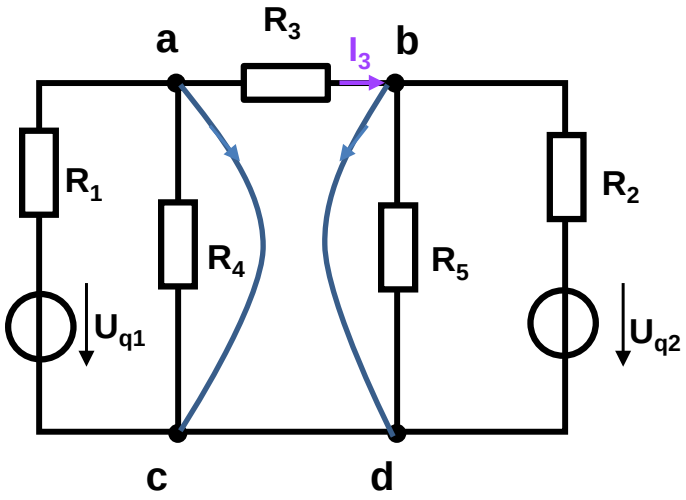
Spannungsquellen kurzschließen, Stromquellen herausnehmen und zwischen a und c bzw. zwischen b und d Widerstand bestimmen:



$$R_{ers1} = R_1 \parallel R_4 = \frac{R_1 \cdot R_4}{R_1 + R_4}$$

$$R_{ers2} = R_2 \parallel R_5 = \frac{R_2 \cdot R_5}{R_2 + R_5}$$

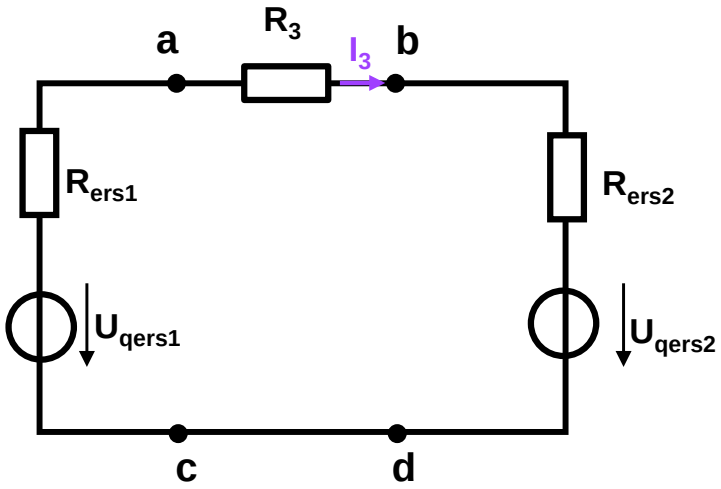
Berechnung von U_{qers}



Kurzschlussstrom zwischen a und c bzw. zwischen b und d bestimmen:

$$I_{k1} = \frac{U_{q1}}{R_1} \qquad I_{k2} = \frac{U_{q2}}{R_2}$$

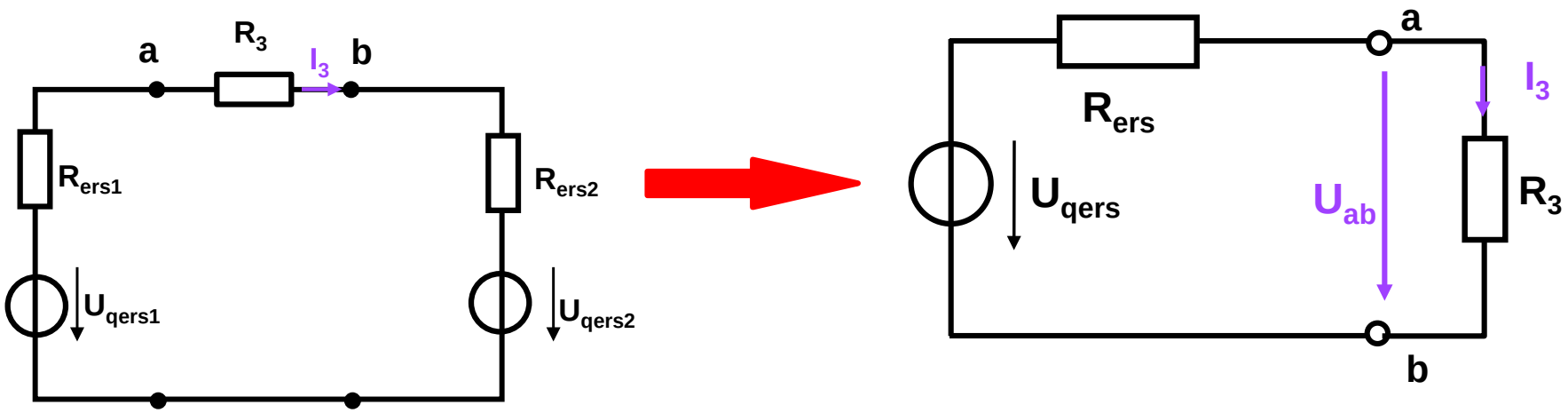
$$U_{qers} = I_k \cdot R_{ers}$$



$$U_{qers1} = I_{k1} \cdot R_{ers1} = \frac{U_{q1} \cdot R_4}{R_1 + R_4}$$

$$U_{qers2} = I_{k2} \cdot R_{ers2} = \frac{U_{q2} \cdot R_5}{R_2 + R_5}$$

$$R_{ers1} = \frac{R_1 \cdot R_4}{R_1 + R_4} \qquad R_{ers2} = \frac{R_2 \cdot R_5}{R_2 + R_5}$$



$$U_{qers} = U_{qers1} - U_{qers2} = \frac{U_{q1} \cdot R_4}{R_1 + R_4} - \frac{U_{q2} \cdot R_5}{R_2 + R_5}$$

$$R_{ers} = R_{ers1} + R_{ers2} = \frac{R_1 \cdot R_4}{R_1 + R_4} + \frac{R_2 \cdot R_5}{R_2 + R_5}$$

$$I_3 = \frac{U_{qers}}{R_{ers} + R_3}$$